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ON THE DISPERSION OF TORSIONAL WAVES IN THE IMPERFECTLY BONDED FINITELY PRE-STRAINED BI-MATERIAL HOLLOW CYLINDER MADE OF HIGHLY-ELASTIC MATERIAL

Tamer KEPÇELER, Eyyüp KARAFAKI*

Yildiz Technical University, Dept. of Mechanical Engineering, Yıldız-ISTANBUL

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ABSTRACT

In this paper, the influence of the bonded imperfectness on torsional wave dispersion in the finitely pre-strained hollow bi-material compound circular cylinder made of highly-elastic material were investigated. The investigations are carried out within the scope of the piecewise homogeneous body model with the use of the Three-dimensional Linearized Theory of Elastic Waves in Initially Stresses Bodies. The mechanical relations of the materials of the cylinders are described through the harmonic potential. Numerical results on the effects of the imperfectness of the boundary condition on the influence of the initial stresses on the wave propagation velocity are presented and discussed.
Keywords: Torsional wave dispersion, imperfect contact condition, initial stress, compounded cylinder.

ELASTİKİYETİ YÜKSEK MALZEMEDEN YAPILMIŞ ÖNGERİLMELİ İDEAL OLMAYAN BAĞLANMAYA SAHİP İÇİ BOŞ BİLEŞİK SİLİNDİRDE BURULMA DALGALARININ YAYILMASI

ÖZET

Bu çalışmada, elastikiyeti yüksek malzemeden yapılmış öngerilmeli içi boş bileşik silindirde burulma dalga yayılımına ideal olmayan bağlanmanın etkisi araştırıldı. Araştırmalar Öngerilmeli Cisimlerde Üç Boyutlu Doğrusallaştırılmış Elastik Dalga Yayılımı Teorisi kullanılarak parçalı homojen cisim modeli çerçevesinde yürütülmektedir. Silindir malzemesinin mekanik ilişkileri harmonik potansiyel ile tanımlanmıştır. Öngerilme ile ideal olmayan sınır şartlarının dalga yayılım hızına etkisinin sayısal sonuçları sunulmuş ve tartışılmıştır.
Anahtar Sözcükler: Burulma dalga yayılımı, ideal olmayan temas koşulları, öngerilme, bileşik silindir.

1. INTRODUCTION

In many cases the control of the adhesion quality in the layered materials is made through measurement of the acoustic wave propagation velocity in these materials. Under these measurement procedures it is necessary to have imformation on the corresponding theoretical results related to the influence of the bonded imperfection on the dispersion of these waves. In connection with this, the investigation carried out in the present paper which relate to the study of the influence of the bonded imperfection on the torsional wave dispersion in the bi-layered

* Corresponding Author/Sorumlu Yazar: e-mail/e-ileti: eyyupkara@yahoo.com, tel: (212) 383 28 42

finitely pre-strained hollow circular cylinder has significant not only in a theoretical sense but also in a practical sense in the corresponding branches of modern engineering.
The subject of the papers [1–2] is the investigation of the dispersion relations of the torsional waves in a pre-stressed compounded cylinder.

The torsional wave propagation in the compounded cylinder (without initial stresses) with an imperfect interface is studied in paper [4]. In [5] the investigations carried out in the papers [1,3] are developed for the case where the contact condition on the interface surface is imperfect. As in [4], the imperfectness of the contact condition is formulated according to the model used in [6]. Moreover, in the present paper, as in [1,3], the mathematical formulations of the corresponding eigen-value problems are made within the scope of the piecewise homogenous body model with the use of the equations and relations of the TLTEWISB. It is assumed that the elasticity relations of the cylinders' materials are given through the Murnaghan Potential [7].

In these works it is assumed that the initial strains in the constituents are small and these strains are calculated within the scope of the classical linear theory of elasticity. The results of the investigations can be employed only for the compounded cylinders made from stiff materials. But these results are not suitable for the compounded cylinders fabricated from the high elastic materials such as elastomers, various type polymers and etc. Therefore in present paper attempt is made for the development of the investigations carried out in the papers [1,3] for the hollow compound cylinder made from high elastic materials, in other words for the case where the initial strains in the components of the cylinder are finite ones and the magnitude of those are not restricted. In this case, as in [3], it is assumed that in each component of the compounded cylinder there exists only the homogenous normal stress acting on the areas which are perpendicular to the lying direction of the cylinders. The mechanical relations of the materials of the cylinders are described through the harmonic potential in the papers [13] and [14].

As in paper [5], in this study the influence of the imperfectness of the contact condition on the torsional wave propagation in the initially stressed (stretched) compounded circular cylinder is investigated. But, as in papers [13] and [14] mechanical relations of the materials of the cylinders are described through the harmonic potential instead of the Murnaghan Potential.

2. FORMULATION OF THE PROBLEM

We consider the compound (composite) circular cylinder shown in Fig. 1 and assume that in the initial state the radius of the internal circle of the inner hollow cylinder is R and the thickness of the inner and outer cylinders are $h^{(1)}$ and $h^{(2)}$, respectively. In the initial state we determine the position of the points of the cylinders by the Lagrangian coordinates in the cylindrical system of coordinates $Orqz$.

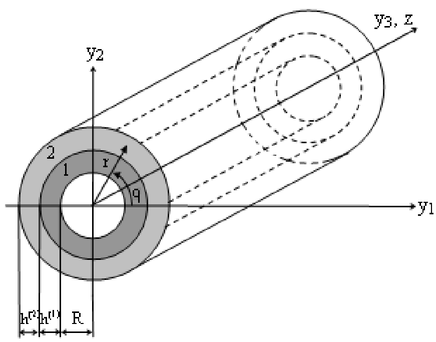


Figure 1. The geometry of the bi-material compound hollow cylinder.

Assume that the cylinders have infinite length in the direction of the Oz axis and the initial stress state in each component of the considered body is axisymmetric with respect to this axis and homogeneous. With the initial state of the cylinders we associate the Lagrangian cylindrical system of coordinates $O'r'q'z'$. The values related to the inner and external hollow cylinders will be denoted by the upper indices (1) and (2), respectively. Furthermore, we denote the values related to the initial state by an additional upper index, 0. Thus, the initial strain state in the inner and external hollow cylinders can be determined as follows.

$$u_m^{(k),0} = (\lambda_m^{(k)} - 1) y_m \qquad \lambda_1^{(k)} = \lambda_2^{(k)} \neq \lambda_3^{(k)} \qquad \lambda_m^{(k)} = const \quad m = 1,2,3 \;\; k = 1,2 \tag{1}$$

where $u_m^{(k),0}$ is a displacement and $l_m^{(k)}$ is the elongation along the axis. We introduce the following notation:

$$y'_i = \lambda_i^{(k)} y_i \qquad r' = \lambda_1^{(k)} r \qquad R' = \lambda_1^{(2)} R \tag{2}$$

The values related to the system of the coordinates associated with the initial state below, i.e. with $O'r'q'z'$ will be denoted by upper prime.

Within this framework, we investigate the axisymmetric torsional wave propagation along the $O'z'$ axis in the considered body by the use of the following field equations.

The equation of motion is:

$$\frac{\partial}{\partial r'} Q'^{(k)}_{r'q} + \frac{\partial}{\partial z'} Q'^{(k)}_{qz} + \frac{1}{r'}(Q'^{(k)}_{r'q} + Q'^{(k)}_{qr'}) = \rho'^{(k)} \frac{\partial^2}{\partial t^2} u'^{(k)}_q \tag{3}$$

The elasticity relations are:

$$Q'^{(k)}_{r'q} = \omega'^{(k)}_{1221} \frac{\partial u'^{(k)}_q}{\partial r'} - \omega'^{(k)}_{1212} \frac{u'^{(k)}_q}{r'} \,, \quad Q'^{(k)}_{q'z'} = \omega'^{(k)}_{1331} \frac{\partial u'^{(k)}_q}{\partial z'} \tag{4}$$

In (3) and (4) through the $Q'^{(k)}_{r'q}$ and $Q'^{(k)}_{q'z'}$ are the perturbation of the components of Kirchhoff stress tensors. $u'^{(k)}_q$ is the perturbation of the components of the displacement vector. ω's are the constants determined through the mechanical constants of the inner and outer cylinders' materials and through the initial stress state. $\rho'^{(k)}$ is the density.

Green's strain tensors with the displacement vector u in the cylindrical coordinates system are:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} + \frac{1}{2}\left(\frac{\partial u_r}{\partial r}\right)^2 + \frac{1}{2}\left(\frac{\partial u_q}{\partial r}\right)^2 + \frac{1}{2}\left(\frac{\partial u_z}{\partial r}\right)^2$$

$$\varepsilon_{qq} = \frac{1}{r}\frac{\partial u_q}{\partial q} + \frac{u_r}{r} + \frac{1}{2r^2}\left(\frac{\partial u_r}{\partial q} - u_q\right)^2 + \frac{1}{2r^2}\left(\frac{\partial u_q}{\partial q} - u_r\right)^2 + \frac{1}{2r^2}\left(\frac{\partial u_z}{\partial q}\right)^2$$

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z} + \frac{1}{2}\left(\frac{\partial u_r}{\partial z}\right)^2 + \frac{1}{2}\left(\frac{\partial u_q}{\partial z}\right)^2 + \frac{1}{2}\left(\frac{\partial u_z}{\partial z}\right)^2$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} + \frac{\partial u_r}{\partial r}\frac{\partial u_r}{\partial z} + \frac{\partial u_q}{\partial r}\frac{\partial u_q}{\partial z} + \frac{\partial u_z}{\partial r}\frac{\partial u_z}{\partial z}\right)$$

$$\varepsilon_{rq}=\frac{1}{2}\left(\frac{\partial u_q}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial q}-\frac{1}{r}u_q+\frac{1}{r}\frac{\partial u_r}{\partial r}\left(\frac{\partial u_r}{\partial q}-u_r\right)+\frac{1}{r}\frac{\partial u_q}{\partial r}\left(\frac{\partial u_q}{\partial q}-u_q\right)+\frac{\partial u_z}{\partial r}\frac{\partial u_z}{\partial q}\right)$$

$$\varepsilon_{qz}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_z}{\partial q}+\frac{\partial u_q}{\partial z}+\frac{1}{r}\frac{\partial u_r}{\partial z}\left(\frac{\partial u_r}{\partial q}-u_r\right)+\frac{1}{r}\frac{\partial u_q}{\partial z}\left(\frac{\partial u_q}{\partial q}-u_q\right)+\frac{1}{r}\frac{\partial u_z}{\partial q}\frac{\partial u_z}{\partial z}\right) \tag{5}$$

Using the expression (1) and (5) we obtain the following initial strains:

$$\varepsilon_{rr}^{(k),0}=\varepsilon_{qq}^{(k),0}=\frac{1}{2}\left(\left(\lambda_1^{(k)}\right)^2-1\right),\quad \varepsilon_{zz}^{(k),0}=\frac{1}{2}\left(\left(\lambda_3^{(k)}\right)^2-1\right)$$

$$\varepsilon_{rq}^{(k),0}=\varepsilon_{rz}^{(k),0}=\varepsilon_{qz}^{(k),0}=0 \tag{6}$$

According to the expression (6), the following relations can be written:

$$\frac{\partial}{\partial\varepsilon_{rr}^{(k),0}}=\frac{\partial}{\partial\varepsilon_{qq}^{(k),0}}=\frac{1}{\lambda_1^{(k)}}\frac{\partial}{\partial\lambda_1^{(k)}},\qquad \frac{\partial}{\partial\varepsilon_{zz}^{(k),0}}=\frac{1}{\lambda_3^{(k)}}\frac{\partial}{\partial\lambda_3^{(k)}} \tag{7}$$

Consider the definition of the stress and strain tensors in the large elastic deformation theory. For this purpose we use the Lagrange coordinates r, q and z in the cylindrical system of coordinates Orqz.

Consider the determination of the Kirchhoff stress tensor. The use of various types of stress tensors in the large (finite) elastic deformation theory is connected with the reference of the components of these tensors to the unit area of the relevant surface elements in the deformed or un-deformed state. This is because, in contrast to the linear theory of elasticity, in the finite elastic deformation theory, the difference between the areas of the surface elements taken before and after deformation must be accounted for in the derivation of the equation of motion and under satisfaction of the boundary conditions. According to the aim of the present investigation, we here consider two types of stress tensors denoted by $\tilde{q}$ and $\tilde{s}$ the components of which refer to the unit area of the relevant surface elements in the un-deformed state, but which act on the surface elements in the deformed state. The physical components $S_{(ij)}$ of the stress tensor $\tilde{s}$ are determined through the strain energy potential $\Phi=\Phi(\varepsilon_{rr},\varepsilon_{qq},.....,\varepsilon_{qz})$ by the use of the following expression:

$$S_{(ij)}=\frac{1}{2}\left(\frac{\partial}{\partial\varepsilon_{(ij)}}+\frac{\partial}{\partial\varepsilon_{(ji)}}\right)\Phi \tag{8}$$

Elasticity relationship of cylinders is expressed by the harmonic potential as in [12].

$$\Phi=\frac{1}{2}\lambda(s_1)^2+\mu s_2 \tag{9}$$

$$s_1=\sqrt{1+2\varepsilon_1}+\sqrt{1+2\varepsilon_2}+\sqrt{1+2\varepsilon_3}-3$$

$$s_2=\left(\sqrt{1+2\varepsilon_1}-1\right)^2+\left(\sqrt{1+2\varepsilon_2}-1\right)^2+\left(\sqrt{1+2\varepsilon_3}-1\right)^2 \tag{10}$$

In relations (8), (9) and (10), λ and μ are material constants and ε_i are the principal values of Green's strain tensor.

Using the expression (9) and (10) we obtain the following expression for the strain energy potential in the initial state:

$$\Phi^{(k),0} = \frac{1}{2}\lambda^{(k)}\left(2\lambda_1^{(k)} + \lambda_3^{(k)} - 3\right)^2 + \mu^{(k)}\left(2\left(\lambda_1^{(k)} - 1\right)^2 + \left(\lambda_3^{(k)} - 1\right)^2\right) \tag{11}$$

Using (10) and (11) we obtain the following expressions for the stresses in the initial state:

$$S_{zz}^{(k),0} = \left[\lambda^{(k)}\left(2\lambda_1^{(k)} + \lambda_3^{(k)} - 3\right) + 2\mu^{(k)}\left(\lambda_3^{(k)} - 1\right)\right]\left(\lambda_3^{(k)}\right)^{-1}$$
$$S_{rq}^{(k),0} = S_{rz}^{(k),0} = S_{zq}^{(k),0} = S_{rr}^{(k),0} = S_{qq}^{(k),0} = 0 \tag{12}$$

The stress tensor $\tilde{q}$ is called the Kirchhoff stress tensor. Using kirchhoff stress tensor in the initial state we obtain:

$$\omega_{1221}'^{(k)} = \omega_{1212}'^{(k)} = \frac{\mu^{(k)}}{\lambda_3^{(k)}},$$
$$\omega_{1331}'^{(k)} = \frac{\lambda_1^{(k)}}{\lambda_1^{(k)} + \lambda_3^{(k)}}\left(2\mu^{(k)} - \lambda^{(k)}\left(2\lambda_1^{(k)} + \lambda_3^{(k)} - 3\right)\right) + \frac{1}{\lambda_3^{(k)}} S_{33}^{(k),0} \tag{13}$$

Torsional wave propagation in the compound hollow cylinder will be investigated by the use of Eqs. (3), (4) and (13) as in [13].

The imperfectness of the contact conditions is identified by discontinuities of the displacements and forces across the mentioned interface. A review of the mathematical modeling of the various type incomplete contact conditions for elastodynamics problems has been detailed in a paper by Martin [8]. It follows from this paper that for most models the discontinuity of the displacement u^+ and force f^+ vectors on one side of the interface are assumed to be linearly related to the displacement u_- and force f_- vectors on the other side of the interface. This statement, as in the paper by Rokhlin and Wang [9], can be presented as follows:

$$[f] = Cu^- + Df^-, \quad [u] = Gu^- + Ff^-, \tag{14}$$

where C, D, G and F are three-dimensional (3 x 3) matrices and the square brackets indicate a jump in the corresponding quantity across the interface. Consequently, if the interface is at

$r = R + h_1$, then

$$[u] = u\Big|_{r=R+h_1+0} - u\Big|_{r=R+h_1-0} \qquad [f] = f\Big|_{r=R+h_1+0} - f\Big|_{r=R+h_1-0}, \tag{15}$$

It follows from (14) that we can write incomplete contact conditions for various particular cases by the selection of the matrices C, D, G and F. One of such selections was made in the paper by Jones and Whitter [6], according to which, it is assumed that C = D = G = 0. In this case it is obtained from (14) that

$$[f] = 0, \quad [u] = Ff^- \tag{16}$$

where F is a constant diagonal matrix. The model (16) simplifies significantly the solution procedure of the corresponding problems and is adequate sufficiently with many real cases. Therefore, this model (i.e. the model (16)) has been used in many investigations carried out by [4], [5], [6], [10], [11], and [15]. According to this statement, we also use the model (16) for the mathematical formulation of the incomplete contact conditions which can be written for the problem under consideration as follows:

$$Q'^{(1)}_{rq}\Big|_{r'=\lambda_2^{(1)}\kappa R}=0$$

$$Q'^{(2)}_{rq}\Big|_{r'=\lambda_2^{(2)}\kappa R\left(1+\frac{h_1}{R}+\frac{h_2}{R}\right)}=0$$

$$Q'^{(1)}_{rq}\Big|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)}=Q'^{(2)}_{rq}\Big|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)}\Rightarrow\ Q'^{(1)}_{rq}\Big|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)}-Q'^{(2)}_{rq}\Big|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)}=0 \tag{17}$$

$$u_q^{(2)}\Big|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)}-u_q^{(1)}\Bigg|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)}=\frac{\mathrm{RF}Q'^{(1)}_{rq}}{\mu_1}$$

The parameter F in (17) characterizes the shear-sprıng type imperfectness between the cylinders under consideration. $F=0$ determines continuous contact condition and $F>0$ determines imperfect contact condition. In this paper, the influence of this parameter F on the dispersion curves are investigated.

3. SOLUTION PROCEDURE AND OBTAINING THE DISPERSION RELATION

As we assume that the harmonic torsional wave propagates along the Oz axis, we can accordingly represent the displacement $u_q^{(m)}(r,z,t)$ as,

$$u'^{(m)}_q(r',z',t)=-\frac{\partial}{\partial r'}\psi^{(m)}(r',z',t) \tag{18}$$

where the function $\psi^{(m)}$ in (18) satisfies the equation written below.

$$\left[\Delta_1+(\xi'^{(m)})^2\frac{\partial^2}{\partial z'^2}-\frac{\rho'}{\omega'_{1221}}\frac{\partial^2}{\partial t^2}\right]\psi=0 \tag{19}$$

where

$$\Delta'_1=\frac{\partial^2}{\partial r'^2}+\frac{1}{r'}\frac{\partial}{\partial r'}\qquad \xi'^{(m)2}_n=\frac{2\lambda_3^{(m)3}}{\lambda_2^{(m)2}\left(\lambda_2^{(m)}+\lambda_3^{(m)}\right)} \tag{20}$$

It follows from the problem statement that the presentation

$$\psi^{(m)}(r',z',t)=\varphi^{(m)}(r')\mathrm{e}^{\mathrm{i}(\kappa z'-\omega t)} \tag{21}$$

holds. Thus, we obtain from (19), (21) the following equation for unknown function $\varphi^{(m)}(r')$

$$\left[\frac{d^2}{dr'^2}+\frac{1}{r'}\frac{d}{dr'}-(\xi'^{(m)})^2\kappa^2+\frac{\lambda_3^{(m)}\rho'^{(m)}\omega^2}{\mu^{(m)}}\right]\varphi(r')\mathrm{e}^{\mathrm{i}(\kappa z'-\omega t)}=0 \tag{22}$$

Introducing the notation

$$(s^{(m)})^2=(\xi'^{(m)})^2\kappa^2+\frac{\lambda_3^{(m)}\rho'^{(m)}\omega^2}{\mu^{(m)}} \tag{23}$$

The solution to the equation (22) can be written as follows.

$$s^{(1)^2} > 0 \Rightarrow \varphi^{(1)}(r') = A^{(1)}\,\mathrm{J}_0\left(s^{(1)}\kappa r'\right) + B^{(1)}\,\mathrm{Y}_0\left(s^{(1)}\kappa r'\right)$$
$$s^{(2)^2} > 0 \Rightarrow \varphi^{(2)}(r') = A^{(2)}\,\mathrm{J}_0\left(s^{(2)}\kappa r'\right) + B^{(2)}\,\mathrm{Y}_0\left(s^{(2)}\kappa r'\right) \tag{24}$$

$$s^{(1)^2} < 0 \Rightarrow \varphi^{(1)}(r') = A^{(1)}\,\mathrm{I}_0\left(s^{(1)}\kappa r'\right) + B^{(1)}\,\mathrm{K}_0\left(s^{(1)}\kappa r'\right)$$
$$s^{(2)^2} < 0 \Rightarrow \varphi^{(2)}(r') = A^{(2)}\,\mathrm{I}_0\left(s^{(2)}\kappa r'\right) + B^{(2)}\,\mathrm{K}_0\left(s^{(2)}\kappa r'\right) \tag{25}$$

Using the equations (4), (18), (21), (24) and (25) we obtain the following dispersion equation from the condition (17).

$$\det\left\|\alpha_{ij}(\mathrm{c}/\,\mathrm{c}_2^{(2)}, \mathrm{kR}, \mathrm{h}_1, \mathrm{h}_2, \mathrm{F}, \lambda_m^{(k)}, \lambda^{(\mathrm{k})}, \mu^k, \rho^{(\mathrm{k})})\right\| = 0, \qquad \mathrm{i;j} = 1,2,3,4 \tag{26}$$

Where

$$\alpha_{11} = \begin{cases} -\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}\left(\mathrm{s}^{(1)}\kappa\right)^2 \mathrm{J}_2\left(\mathrm{s}^{(1)}\kappa r'\right) & , \mathrm{s}^{(1)^2} > 0 \\ -\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}\left(\mathrm{s}^{(1)}\kappa\right)^2 \mathrm{I}_2\left(\mathrm{s}^{(1)}\kappa r'\right) & , \mathrm{s}^{(1)^2} < 0 \end{cases}$$

$$\alpha_{12} = \begin{cases} -\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}\left(\mathrm{s}^{(1)}\kappa\right)^2 \mathrm{Y}_2\left(\mathrm{s}^{(1)}\kappa r'\right) & , \mathrm{s}^{(1)^2} > 0 \\ -\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}\left(\mathrm{s}^{(1)}\kappa\right)^2 \mathrm{K}_2\left(\mathrm{s}^{(1)}\kappa r'\right) & , \mathrm{s}^{(1)^2} < 0 \end{cases}$$

$$\alpha_{13} = \alpha_{14} = \alpha_{21} = \alpha_{22} = 0$$

$$\alpha_{23} = \begin{cases} -\dfrac{\mu^{(2)}}{\lambda_3^{(2)}}\left(\mathrm{s}^{(2)}\kappa\right)^2 \mathrm{J}_2\left(\mathrm{s}^{(2)}\kappa r'\right) & , \mathrm{s}^{(2)^2} > 0 \\ -\dfrac{\mu^{(2)}}{\lambda_3^{(2)}}\left(\mathrm{s}^{(2)}\kappa\right)^2 \mathrm{I}_2\left(\mathrm{s}^{(2)}\kappa r'\right) & , \mathrm{s}^{(2)^2} < 0 \end{cases}$$

$$\alpha_{24} = \begin{cases} -\dfrac{\mu^{(2)}}{\lambda_3^{(2)}}\left(\mathrm{s}^{(2)}\kappa\right)^2 Y_2\left(\mathrm{s}^{(2)}\kappa r'\right) & , \mathrm{s}^{(2)^2} > 0 \\ -\dfrac{\mu^{(2)}}{\lambda_3^{(2)}}\left(\mathrm{s}^{(2)}\kappa\right)^2 \mathrm{K}_2\left(\mathrm{s}^{(2)}\kappa r'\right) & , \mathrm{s}^{(2)^2} < 0 \end{cases}$$

$$\alpha_{31} = \begin{cases} -\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}\left(\mathrm{s}^{(1)}\right)^2 \mathrm{J}_2\left(\mathrm{s}^{(1)}\kappa r'\right) & , \mathrm{s}^{(1)^2} > 0 \\ -\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}\left(\mathrm{s}^{(1)}\right)^2 \mathrm{I}_2\left(\mathrm{s}^{(1)}\kappa r'\right) & , \mathrm{s}^{(1)^2} < 0 \end{cases}$$

$$\alpha_{32}=\begin{cases}-\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}\left(s^{(1)}\right)^2 Y_2\left(s^{(1)}\kappa r'\right) & , s^{(1)^2}>0\\ -\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}\left(s^{(1)}\right)^2 K_2\left(s^{(1)}\kappa r'\right) & , s^{(1)^2}<0\end{cases}$$

$$\alpha_{33}=\begin{cases}\dfrac{\mu^{(2)}}{\lambda_3^{(2)}}\left(s^{(2)}\right)^2 J_2\left(s^{(2)}\kappa r'\right) & , s^{(2)^2}>0\\ \dfrac{\mu^{(2)}}{\lambda_3^{(2)}}\left(s^{(2)}\right)^2 I_2\left(s^{(2)}\kappa r'\right) & , s^{(2)^2}<0\end{cases}$$

$$\alpha_{34}=\begin{cases}\dfrac{\mu^{(2)}}{\lambda_3^{(2)}}\left(s^{(2)}\right)^2 Y_2\left(s^{(2)}\kappa r'\right) & , s^{(2)^2}>0\\ \dfrac{\mu^{(2)}}{\lambda_3^{(2)}}\left(s^{(2)}\right)^2 K_2\left(s^{(2)}\kappa r'\right) & , s^{(2)^2}<0\end{cases}$$

$$\alpha_{41}=\begin{cases}\left[\ \underbrace{J_1\left(s^{(1)}\kappa r'\right)}-\dfrac{RF}{\lambda_3^{(1)}}\left(s^{(1)}\kappa\right)J_2\left(s^{(1)}\kappa r'\right)\ \right] & , s^{(1)^2}>0\\ \left[\ \underbrace{-I_1\left(s^{(1)}\kappa r'\right)}-\dfrac{RF}{\lambda_3^{(1)}}\left(s^{(1)}\kappa\right)I_2\left(s^{(1)}\kappa r'\right)\ \right] & , s^{(1)^2}<0\end{cases}$$

$$\alpha_{42}=\begin{cases}\underbrace{\left[Y_1\left(s^{(1)}\kappa r'\right)-\dfrac{RF}{\lambda_3^{(1)}}\left(s^{(1)}\kappa\right)Y_2\left(s^{(1)}\kappa r'\right)\right]}_{s^2>0\ \Rightarrow\ \alpha_{42}} & , s^{(1)^2}>0\\ \left[\underbrace{\underbrace{K_1\left(s^{(1)}\kappa r'\right)}_{0}-\dfrac{RF}{\lambda_3^{(1)}}\left(s^{(1)}\kappa\right)K_2\left(s^{(1)}\kappa r'\right)}_{s^2<0\ \Rightarrow\ \alpha_{42}}\right] & , s^{(1)^2}<0\end{cases}$$

$$\alpha_{43}=\begin{cases}-s^{(2)}J_1\left(s^{(2)}\kappa r'\right) & , s^{(2)^2}>0\\ s^{(2)}I_1\left(s^{(2)}\kappa r'\right) & , s^{(2)^2}<0\end{cases}$$

$$\alpha_{44}=\begin{cases}-s^{(2)}Y_1\left(s^{(2)}\kappa r'\right) & , s^{(2)^2}>0\\ -s^{(2)}K_1\left(s^{(2)}\kappa r'\right) & , s^{(2)^2}<0\end{cases} \tag{27}$$

Thus, the dispersion equation for the considered torsional wave propagation problem has been derived in the form presented in (27).

4. NUMERICAL RESULTS AND DISCUSSIONS

We have found that the first lowest mode which is non-dispersive homogenous hollow cylinder, becomes dispersive for a compound one. The limiting value of the torsional wave speed for the case considered is determined from dispersion equation (26), (27) by using power series expansions of Bessel functions, retaining only the dominant term as $k\mathrm{R} \to 0$:

$$\frac{c}{c_2^{(1)}} = \left[\frac{\frac{\mu^{(1)}}{\lambda_3^{(1)}}\left(\xi_1^{,(1)}\right)^2 + \frac{\mu^{(2)}}{\lambda_3^{(2)}}\frac{\left(\zeta_2^{\,4}-\zeta_1^{\,4}\right)}{\left(\zeta_1^{\,4}-1\right)}\left(\xi_1^{,(2)}\right)^2}{\mu^{(1)}+\mu^{(2)}\frac{c_2^{(1)2}}{c_2^{(2)2}}\frac{\left(\zeta_2^{\,4}-\zeta_1^{\,4}\right)}{\left(\zeta_1^{\,4}-1\right)}} \right]^{1/2}$$

$$\frac{c}{c_2^{(1)}} = \left[\frac{\frac{\mu}{\lambda_3^{(1)}}\left(\xi_1^{,(1)}\right)^2 + \frac{1}{\lambda_3^{(2)}}\alpha\left(\xi_1^{,(2)}\right)^2}{\mu+\mu\alpha\mathrm{RO}} \right]^{1/2} \tag{28}$$

Where

$$\alpha = \frac{\left(\zeta_2^{\,4}-\zeta_1^{\,4}\right)}{\left(\zeta_1^{\,4}-1\right)} \qquad \zeta_1 = 1+\frac{h1}{R} \qquad \zeta_2 = 1+\frac{h1}{R}+\frac{h2}{R} \tag{29}$$

In the case where $\lambda_3^{(m)} = \lambda_2^{(m)} = 1.0$, the expression (28) transforms to the following one.

$$\left(\frac{c}{c_2^{(1)}}\right)^2 = \frac{\mu^{(1)}+\mu^{(2)}\alpha}{\mu^{(1)}+\mu^{(2)}\alpha\frac{c_2^{(1)2}}{c_2^{(2)2}}} \tag{30}$$

Moreover, the expression (28) is a generalization of the corresponding one attained in the paper [1] for the finite initial strain state. Note that in the paper [1] this type expression was obtained for the small initial strain state.

It follows from the expression (28) and (20) that the limit values of $\mathrm{c}/\mathrm{c}_2^{(1)}$, where $\mathrm{c}_2^{(1)} = \sqrt{\mu^{(1)}/\rho^{(1)}}$ decrease with $\mu^{(1)}/\mu^{(2)}$ and increase with $\lambda = (\lambda_3^{(1)} = \lambda_3^{(2)})$ Consequently, the initial stretching (compression) of the compound cylinder along the torsional wave propagation direction causes to increase (to decrease) of the limit velocity of this wave as $kR \to 0$. According to the known physical-mechanical consideration, the other limit value of the velocity of the considered wave, i.e. the limit velocity as $kR \to \infty$ must be equal to min $\left\{c_2^{(1)}(\lambda_3^{(1)}), c_2^{(2)}(\lambda_3^{(2)})\right\}$,i.e. the following relation must be hold.

$$c \to \min\left\{c_2^{(1)}(\lambda_3^{(1)}), c_2^{(2)}(\lambda_3^{(2)})\right\} \text{ as } kR \to \infty \tag{31}$$

Accuracy of the algorithms used in the considered problem for a similar situation in the [13] paper has proven. Thereafter, $\mathrm{c}/\mathrm{c}_2^{(1)}$ and kR have been studied.

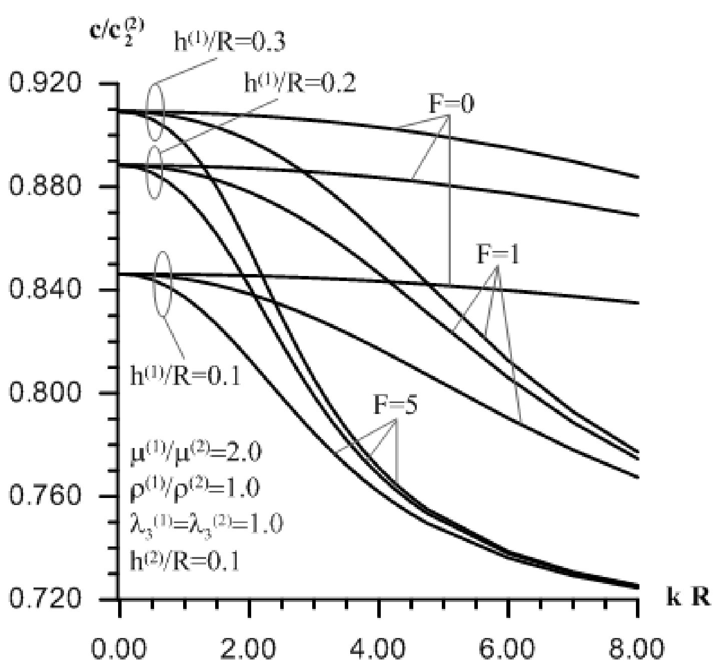


Figure 2. The influence of the parameter F on dispersion curves under varios $h^{(1)}/R$.

Assuming that the inner cylinder is harder material $\mu^{(1)}/\mu^{(2)}=2$ and there is no pretensioning in both cylinder $\lambda_3^{(1)}=\lambda_3^{(2)}=1$, together with the change in thicknesses the influence of the parameter F was investigated in Fig. 2 and Fig 3.

As seen in Fig. 2 wave propagation velocity decreases with the increase of $h^{(1)}/R$ and in parallel with the increase in the value of the contacting simulating F wave propagation speed is relatively decreases. In figure 3, with the decrease of $h^{(2)}/R$, wave propagation velocity and the influence of the parameter F increases.

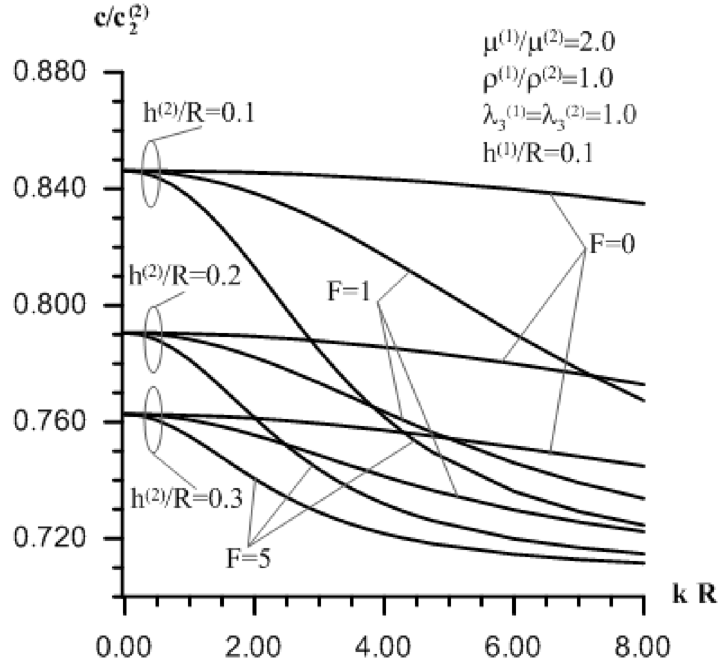


Figure 3. The influence of the parameter F on dispersion curves under varios $h^{(2)}/R$.

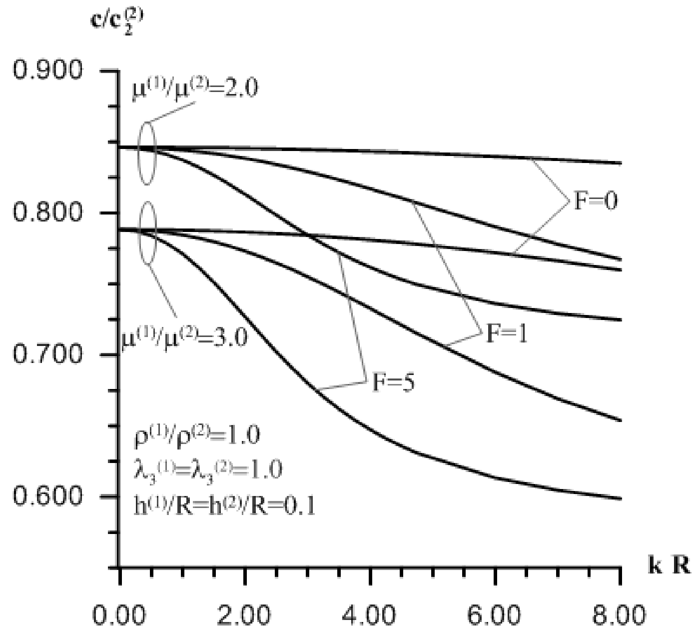


Figure 4. The influence of the parameter F on dispersion curves under higher varios $\mu^{(1)}/\mu^{(2)}$.

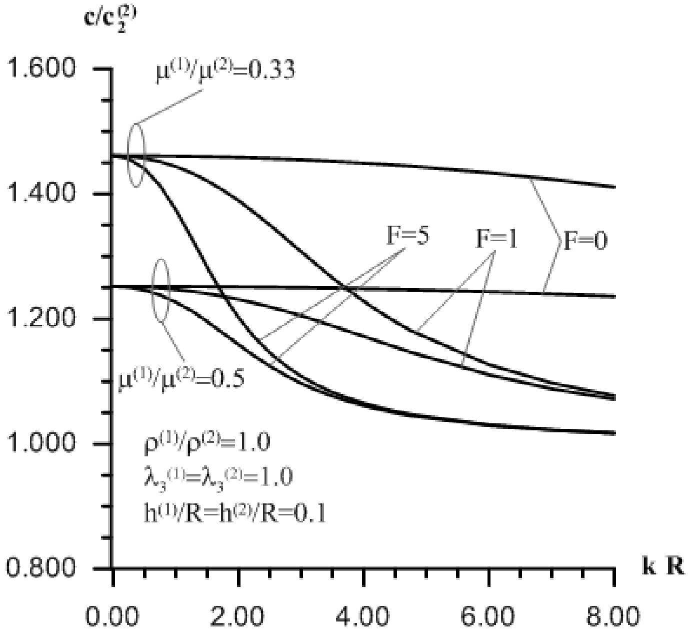
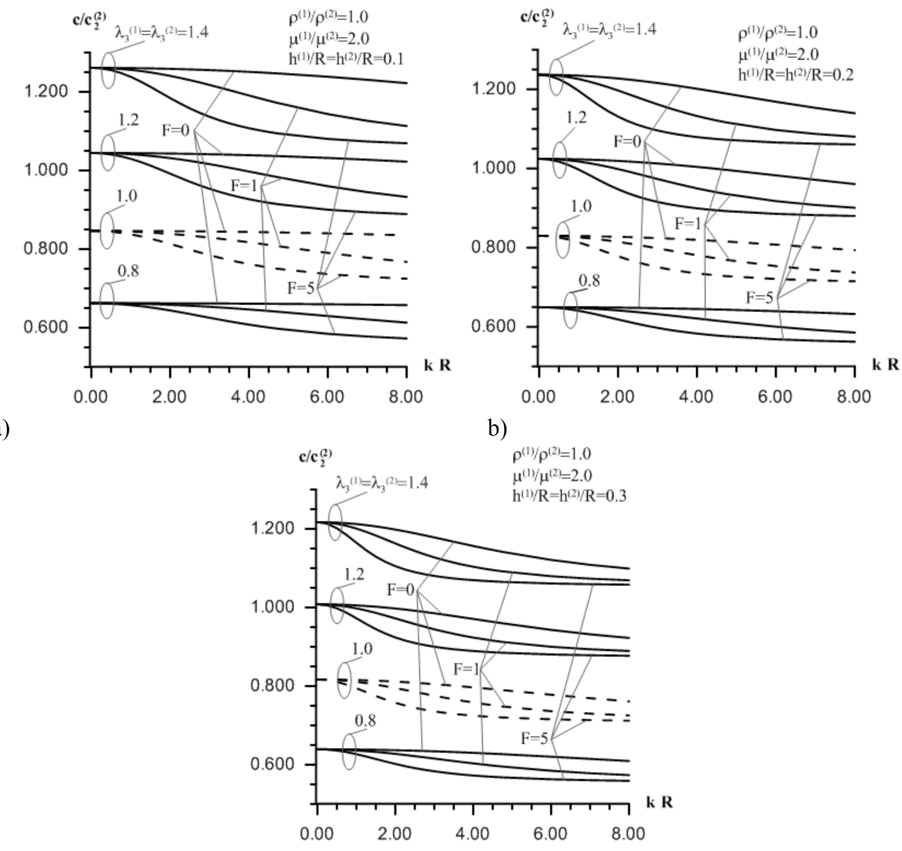


Figure 5. The influence of the parameter F on dispersion curves under lower varios $\mu^{(1)} / \mu^{(2)}$.

Assuming that cylinder thickness were the same $h^{(1)} / R = h^{(2)} / R = 0.1$ and there is no pretensioning in both cylinder $\lambda_3^{(1)} = \lambda_3^{(2)} = 1$, together with the change in hardnesses the influence of the parameter F was investigated in Fig. 4 and Fig 5. With the hardening of the both inner and outer cylinder, the influence of the parameter F decreases. In the case where $kR \to 0$, the limit velocity of the wave propagation is the same and independent of the parameter F. Effects of parameter F decreases with kR. Finally, we note that the wave propagation velocity approch to the $\min\left\{c_R^{(1)}, c_R^{(2)}\right\}$ as $kR \to \infty$, where $c_R^{(m)} (m = 1{,}2)$ is a Rayleigh wave velocity of the m-th material.



a) b) c)

Figure 6. The influence of the parameter F on dispersion curves under varios values of the initial strains. a) $h^{(1)}/R = h^{(2)}/R = 0.1$ b) $h^{(1)}/R = h^{(2)}/R = 0.2$ c) $h^{(1)}/R = h^{(2)}/R = 0.3$

The influence of the parameter F on dispersion curves under varios values of the initial strains and thickness were investigated in Fig. 6. We see that with the growing of values of the initial strains the influence of the parameter F becomes more prominent. With the increase of the thickness the influence of the parameter F decreases.

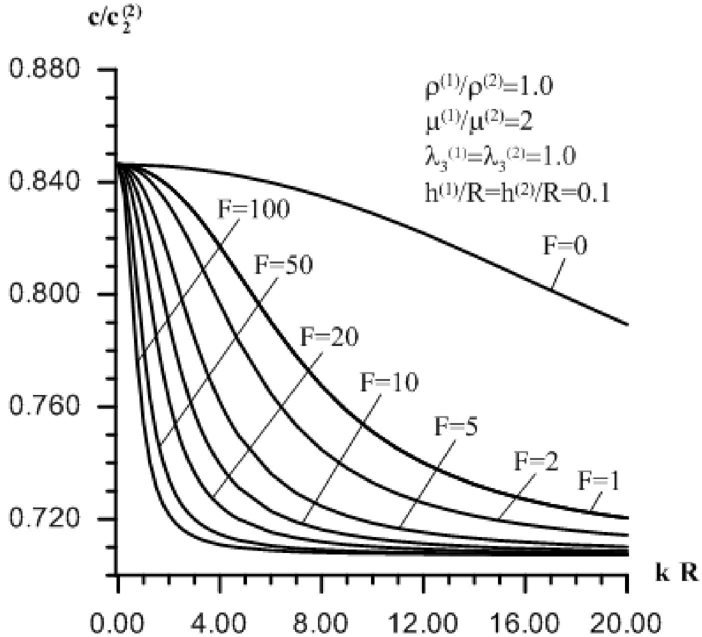


Figure 7. The influence of the parameter F on dispersion curves.

Fig. 7 shows that the imperfectness of the contact condition causes the velocity of the wave propagation to reduce. Note that similar results are also obtained in papers [4] and [5].

5. CONCLUSION

Wave propagation velocity decreases with the increase of $h^{(1)}/R$ and in parallel with the increase in the value of the contacting simulating F, wave propagation speed is relatively decreases. With the decrease of $h^{(2)}/R$, wave propagation velocity and the influence of the parameter increases. With the hardening of the both inner and outer cylinder, the influence of the parameter F decreases. With the growing of values of the initial strains, the influence of the parameter F becomes more prominent. The imperfectness of the contact condition causes the velocity of the wave propagation to reduce.

According to the foregoing results, it can be concluded that the considered type imperfection causes to decrease of the wave propagation velocity and the initial stretching acts significantly not only the wave propagation velocity, but also the magnitude of the mentioned influence of the imperfection of the contact on this velocity.

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