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POWER OPTIMIZATION OF AN IRREVERSIBLE REGENERATIVE BRAYTON CYCLE WITH ISOTHERMAL HEAT ADDITION

***Rajesh Kumar**
Department of Mechanical Engineering,
Amity University Haryana,
Gurgaon-122413, India

S. C. Kaushik
Centre for Energy Studies,
Indian Institute of Technology, Delhi,
New Delhi-110016, India

Raj Kumar
Department of Mechanical Engineering,
YMCA University of Science & Technology,
Faridabad-121006, India

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** Corresponding author: Rajesh Kumar, E-mail: rajesharora1219@rediffmail.com*

ABSTRACT

An irreversible regenerative Brayton cycle model with two heat additions is analyzed here. The external irreversibilities due to finite temperature difference and internal irreversibilities due to fluid friction losses in compressor / turbine, regenerative heat loss, pressure loss are included in the analysis. Power output of the model is obtained and thermodynamically optimized. A detailed analysis shows that with judicious selection of parameters viz. efficiency of turbine and compressor, effectiveness of various heat exchangers, isothermal pressure drop ratio, pressure drop recovery coefficients and heat capacitance rate of the working fluid, the power output of the model can be made to reach its highest possible value. It is well proven with the obtained results that induction of two heat additions significantly enhances model efficiency above 20% as compared to conventional gas power plants. The power output remains constant while the corresponding thermal efficiency increases as regenerator side effectiveness is increased. This meticulous result is different from those obtained by previous researchers. The model analyzed in this paper gives lower values of power output and corresponding thermal efficiency as expected and replicates the results of an irreversible regenerative Brayton cycle model discussed in the literature at pressure recovery coefficients of $\alpha_1=\alpha_2=1$.

1. INTRODUCTION

The most important criterion in the design of real gas power plant is not only efficiency but power output also. Leff [2] analyzed an endoreversible Brayton heat engine following Curzon and Ahlborn [1] and observed the change in Brayton cycle temperatures while altering maximum work in the cycle. Wu and Kiang [3] optimized power output of a Brayton cycle using finite time thermodynamics. Wu [4] optimized the power of an endoreversible Brayton gas heat engine. Wu & Kiang [5] integrated real compression and expansion in Brayton heat engine and found that engine power and engine efficiency are strong functions of the compressor and turbine efficiencies. Ibrahim et al. [6] performed power optimization for a closed ideal Brayton cycle in context with various boundary configurations. Cheng et al. [7] performed power optimization of an endoreversible regenerative Brayton cycle and observed decrease in maximum power and corresponding efficiency with the application of regenerator. Wu et al. [8] investigated performance of a regenerative Brayton heat engine and found that maximum non-dimensional power output of cycle increases from 0.2 to 0.4 by altering regenerator effectiveness between 0.6 and 1. Chen et al. [9] further assessed performance of regenerative Brayton cycle and found that regenerative effectiveness is the deciding factor while calculating power output of the cycle. Many other researchers [10-13] assessed regenerative Brayton cycle based on endoreversible and irreversible configuration with the application of isothermal heat additions in the view of finite time thermodynamic approach. Wang et al. [14] applied the hypothesis of finite time thermodynamics to analyze an irreversible closed intercooled regenerated Brayton cycle and optimized the intercooler pressure ratio for optimum power and corresponding efficiency. Kaushik et al. [15] performed a thermodynamic analysis of an irreversible regenerative Brayton cycle with isothermal heat addition and optimized the power output in context with working medium temperature. They observed an improvement of 15% in the thermal efficiency of Brayton cycle with heat addition at constant temperature. Chen et al. [16] analyzed power and efficiency of an endoreversible closed intercooled regenerated Brayton cycle in the view of finite time thermodynamics. Wang et al. [17-19] performed power optimization by altering effectiveness of various heat exchangers for intercooled and regenerated Brayton cycles coupled to fixed [17] and finite temperature [18-19] heat reservoirs based on endoreversible [17,18] and irreversible [19] mode.. Jubeh [20] performed exergy analysis of a regenerative Brayton cycle and found appreciable increase in second law efficiency at lesser pressure ratio, small environment temperature and elevated entrance temperature of expander with the introduction of two heat additions. Further, Wang et al. [21] investigated power and power density of externally irreversible Brayton cycle with two heat additions in the view of finite time thermodynamics and found the range of isothermal heat addition on various performance parameters of endoreversible Brayton cycle. On the basis of recent literature, a model of an irreversible regenerative Brayton cycle with pressure drop as supplementary irreversibility is considered in this paper and expressions for maximum power output and corresponding thermal efficiency of an irreversible regenerative Brayton cycle are obtained. The effect of effectiveness of various heat exchangers, efficiency of turbine and compressor, heat capacitance rates, isothermal pressure drop ratio and pressure recovery coefficients have been studied in detail and the results are presented on graphs. The model analyzed in this paper gives lower values of power output and corresponding thermal efficiency as expected.

2. THERMODYNAMIC ANALYSIS

An irreversible regenerative Brayton cycle model coupled with a heat source and heat sink of finite heat capacity is shown on T-S diagram in Fig. 1. In this model, state 1 is the entry point of working medium at compressor and compressed up to state 2. Then the working medium enters the regenerator where its partial heating up to state 2R is done by the turbine exhaust. The working medium next enters the hot side heat exchanger with a pressure drop which is reflected using pressure recovery coefficient, $\alpha_1 = p_3/p_2$ and heated up to state 3. First heat addition (Q_H) takes place at constant pressure in hot side heat exchanger while the heat source temperature decreases from T_{H1} to T_{H2}. Again, second heat addition (Q_{H1}) takes place at constant temperature during process 3-4 and heat source temperature decreases from T_{H3} to T_{H4}.The working medium now enters the turbine and expands up to state 5. After expansion, the working medium enters the regenerator to transfer heat partly and then enters the cold side heat exchanger with a pressure drop which is reflected using another pressure recovery coefficient, $\alpha_2 = p_1/p_5$. The working medium is cooled up to state 1, while the heat sink temperature increases from T_{L1} to T_{L2}. Therefore, we consider the closed Brayton cycle model 1-2-2R-3-4-5-5R-1 with real compression / expansion processes and pressure drop irreversibilities for finite heat capacity of external reservoirs. Process (1-2s) and process (4-5s) are isentropic in nature as shown by dotted lines in Figure 1.

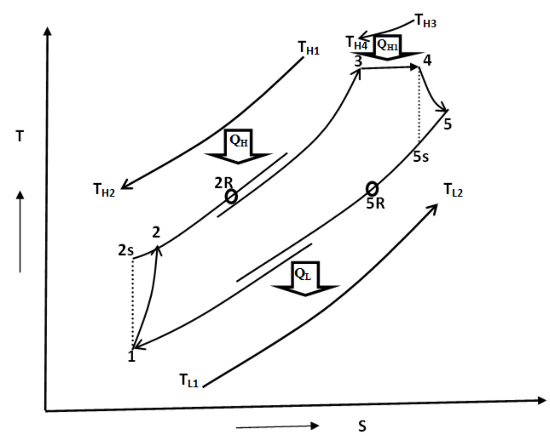

Fig. 1 T-S diagram for irreversible regenerative Brayton heat cycle with isothermal heat addition

The various heat transfer rates are calculated as:

$$Q_H = U_H A_H (LMTD)_H = C_H (T_{H1} - T_{H2}) \quad (1)$$

$$Q_{H1} = U_{H1} A_{H1} (LMTD)_{H1} = C_{H1} (T_{H3} - T_{H4}) \quad (2)$$

$$Q_L = U_L A_L (LMTD)_L = C_L (T_{L2} - T_{L1}) \quad (3)$$

$$Q_R = U_R A_R (LMTD)_R = C_W (T_{2R} - T_2) \quad (4)$$

where,

$$(LMTD)_H = \frac{(T_{H1}-T_3)-(T_{H2}-T_{2R})}{\ln\{(T_{H1}-T_3)/(T_{H2}-T_{2R})\}} \quad (5)$$

$$(LMTD)_{H1} = \frac{(T_{H3}-T_3)-(T_{H4}-T_3)}{\ln\{(T_{H3}-T_3)/(T_{H4}-T_3)\}} \quad (6)$$

$$(LMTD)_L = \frac{(T_{5R}-T_{L2})-(T_1-T_{L1})}{\ln\{(T_{5R}-T_{L2})/(T_1-T_{L1})\}} \quad (7)$$

$$(LMTD)_R = \frac{(T_5-T_{2R})-(T_{5R}-T_2)}{\ln\{(T_5-T_{2R})/(T_{5R}-T_2)\}} \quad (8)$$

From equations (1) to (6),

$$Q_H = \varepsilon_H C_{H,\min}(T_{H1}-T_{2R}) = C_W(T_3-T_{2R}) \quad (9)$$

$$Q_{H1} = \varepsilon_{H1} C_{H1,\min}(T_{H3}-T_3) \quad (10)$$

$$Q_L = \varepsilon_L C_{L,\min}(T_{5R}-T_{L1}) = C_W(T_{5R}-T_1) \quad (11)$$

$$Q_R = \varepsilon_R C_W(T_5-T_2) = C_W(T_5-T_{5R}) \quad (12)$$

where ε_H, ε_{H1}, ε_L and ε_R are the effectiveness of the isobaric heat source side, isothermal heat source side, sink side and regenerative side heat exchangers respectively and can be presented as:

$$\varepsilon_H = \frac{1-e^{-N_H(1-C_{H,\min}/C_{H,\max})}}{1-\frac{C_{H,\min}}{C_{H,\max}}e^{-N_H(1-C_{H,\min}/C_{H,\max})}} \quad (13)$$

$$\varepsilon_L = \frac{1-e^{-N_L(1-C_{L,\min}/C_{L,\max})}}{1-\frac{C_{L,\min}}{C_{L,\max}}e^{-N_L(1-C_{L,\min}/C_{L,\max})}} \quad (14)$$

$$\varepsilon_R = \frac{N_R}{1+N_R} \quad (15)$$

and $\varepsilon_{H1} = \frac{N_{H1}}{1+N_{H1}}$ (16)

The various heat transfer rates and number of transfer units are calculated as:

$$C_{H,\min} = \min(C_H, C_W)$$

$$C_{H1,\min} = \min(C_{H1}, C_W)$$

$$C_{H,\max} = \max(C_H, C_W)$$

$$C_{H1,\max} = \max(C_{H1}, C_W)$$

$$C_{L,\min} = \min(C_L, C_W)$$

$$C_{L,\max} = \max(C_L, C_W)$$

$$N_L = \frac{U_L A_L}{C_{L,\min}}$$

$$N_H = \frac{U_H A_H}{C_{H,\min}}$$

$$N_{H1} = \frac{U_{H1} A_{H1}}{C_{H1,\min}}$$

$$N_R = \frac{U_R A_R}{C_W}$$

The compressor and turbine efficiencies are calculated as:

$$\eta_c = \frac{T_{2s}-T_1}{T_2-T_1} \quad (17)$$

$$\eta_t = \frac{T_4-T_5}{T_4-T_{5s}} \quad (18)$$

Now from equations (7) to (14),

$$T_{5R} = (1-\varepsilon_R)T_5 + \varepsilon_R T_2 \quad (19)$$

$$T_{2R} = (1-\varepsilon_R)T_2 + \varepsilon_R T_5 \quad (20)$$

$$T_1 = (1-b)T_{5R} + bT_{L1} \quad (21)$$

$$T_3 = (1-a)T_{2R} + aT_{H1} \quad (22)$$

$$T_{2s} = (1-\eta_c)T_1 + T_2\eta_c \quad (23)$$

$$T_{4s} = (1-\eta_t^{-1})T_3 + T_5\eta_t^{-1} \quad (24)$$

From second law of thermodynamics for a given model,

$$T_1T_3 = \beta T_{2s}T_{5s} \quad (25)$$

where $\beta = (\chi_t\alpha_1\alpha_2)^{\frac{k-1}{k}}$ and χ_t = p_4/p_3, is the isothermal pressure drop ratio.

Putting the values of T_1, T_3, T_{2s} and T_{5s} from equations (21) - (24) into equation (25), we obtain the quadratic equation in T_2 as:

$$XT_2^2 + YT_2 + Z = 0 \quad (26)$$

Parameters X, Y and Z are listed in Appendix-I. Solution of quadratic equation (26) is written as:

$$T_2 = \frac{-Y+\sqrt{Y^2-4XZ}}{2X} \quad (27)$$

From the first law of thermodynamics, we have:

$$P = Q_H + Q_{H1} - Q_L$$
$$= \varepsilon_H C_{H,\min}(T_{H1}-T_{2R}) + \varepsilon_{H1}C_{H1,\min}(T_{H3}-T_3)$$
$$-\varepsilon_L C_{L,\min}(T_{5R}-T_{L1}) \quad (28)$$

Putting the value of various temperatures into equation (28), we get:

$$P = z_6 - x_7T_2 - y_7T_5 \quad (29)$$

$$\eta = \frac{P}{Q_H + Q_{H1}} = \frac{z_6 - x_7 T_2 - y_7 T_5}{z_7 - x_8 T_2 - y_8 T_5} \quad (30)$$

Parameters x_7, x_8, y_7, y_8, z_6 and z_7 are listed in Appendix-I.

Thus, optimizing equation (29) with respect to T_5 i.e. $\frac{\partial P}{\partial T_5} = 0$ and solution of this equation as:

$$X_1 T_5^2 + Y_1 T_5 + Z_1 = 0 \quad (31)$$

Parameters X_1, Y_1 and Z_1 are recorded in Appendix-I.

Solving equation (31) for T_5, we get the optimum value of T_5 as

$$T_{5,opt} = \frac{-Y_1 - \sqrt{Y_1^2 - 4X_1 Z_1}}{2X_1} \quad (32)$$

3. RESULTS AND DISCUSSION

In order to have mathematical approval of outcome, the effects of various performance parameters viz. efficiency of turbine and compressor, effectiveness of various heat exchangers, isothermal pressure drop ratio, pressure drop recovery coefficients and heat capacitance rate of the working fluid on an irreversible regenerative Brayton heat engine model are investigated. Each one of above mentioned parameter is examined by keeping rest parameters constant as $\varepsilon_H = \varepsilon_{H1} = \varepsilon_L = \varepsilon_R = 0.75$, $T_{H1}=1000$, $T_{H3}=1250$ K, $T_{L1}=300$ K, $\eta_t = \eta_c = 0.8$, $C_W = 1.05$ kWK^{-1}, $C_H = C_{H1} = C_L = 1$ kWK^{-1}, $U_H = U_{H1} = U_L = U_R = 2.0$ kWK^{-1}m^{-2}, $\chi_t = 0.8$, $\alpha_1 = \alpha_2 = 0.95$. The obtained results are presented on graphs and discussed in detail as follows:

3.1 Effect of ε_H, ε_{H1}, ε_L and ε_R

The variations of various effectiveness on power output and corresponding thermal efficiency are shown in figures 2(a) to 2(b). It is clearly seen from these results that maximum power output and corresponding thermal efficiency increases as the effectiveness on isothermal heat source side (ε_{H1}), heat sink side (ε_L) is increased while all the performance parameters decreases as isobaric heat source side effectiveness (ε_H) is increased. It is also found that the power output remains constant while the corresponding thermal efficiency increases as regenerator side effectiveness is increased. The results obtained can also be correlated with heat transfer area. It is required to increase the heat transfer area as the effectiveness is increased which results in increase of cost of the system. So, judicious selection of effectiveness of various heat exchangers is required. However, in general, the variations of various performance parameters with respect to effectiveness are not linear and the relation $\varepsilon_L > \varepsilon_{H1} > \varepsilon_H$ is observed for better execution of the model.

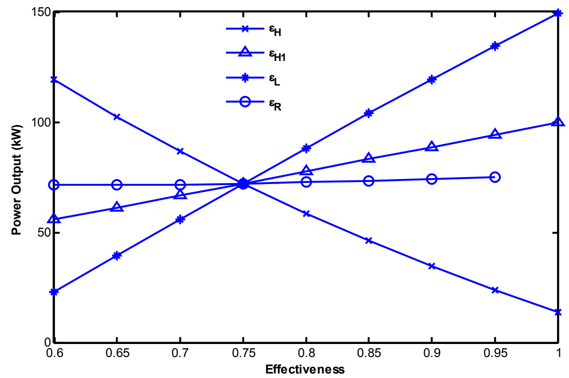

Fig. 2 (a) Variations of Power Output with respect to effectiveness of heat exchangers

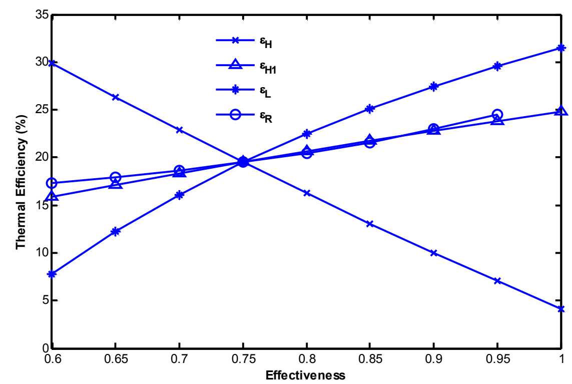

Fig 2(b) Variations of Thermal Efficiency with respect to effectiveness of heat exchangers

3.2 Effect of heat capacitance rates (C_H, C_{H1}, C_L and C_W)

The variations of various heat capacitance rates on maximum power output and corresponding thermal efficiency are shown in figures 3(a) to 3(b). It is clearly observed from these results that maximum power output and thermal efficiency increases with increase in heat capacitance rates of constant temperature source side and sink side reservoirs whereas all the performance parameters shows steep fall with the increase in heat capacitance rate of constant pressure heat source reservoir and cycle working fluid. It is also found that sink side heat capacitance rate is more dominant than constant temperature source side on all the performance parameters of the cycle. In general, the variations of various performance parameters with respect to heat capacitance rates are not linear and the relation $C_L >$

$C_{H1} > C_W$ is observed for better execution of the model. Moreover, the heat capacitance rate of constant pressure heat reservoir (C_H) should be as small as possible.

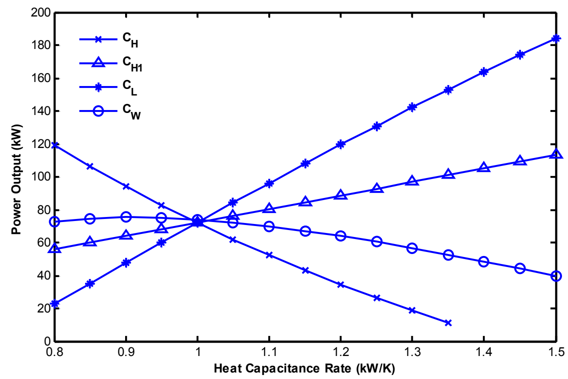

Fig. 3 (a) Variations of Power Output with respect to heat capacitance rates

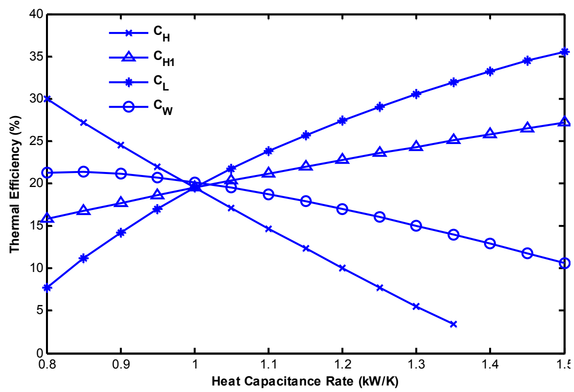

Fig. 3 (b) Variations of Thermal Efficiency with respect to heat capacitance rates

3.3 Effects of turbine and compressor efficiencies (η_t and η_c)

The variations of turbine and compressor efficiencies on power output and corresponding thermal efficiency of an irreversible regenerative Brayton heat engine cycle with finite capacity heat reservoir are shown in figures 4(a) to 4(b). It is seen from these figures that maximum power output and thermal efficiency increases with the increase in component efficiencies (η_t and η_c) which indicates that larger the component efficiency is, better the performance of the cycle. It is also found that turbine efficiency (η_t) adds more effect on the thermodynamic performance of an irreversible regenerative Brayton heat engine cycle than the compressor efficiency (η_c). Consequently, for real Brayton heat engine cycle, lots of research and investigation is still required on compressor efficiency.

3.4 Effects of isothermal pressure drop ratio

Figure 5 shows the effect of isothermal pressure ratio on various performance parameters of an irreversible regenerative Brayton heat engine cycle. It is seen from these figures that maximum power output and corresponding thermal efficiency increases as isothermal pressure drop ratio is increased. It is also seen from these figures that various performance parameters attains their maximum value at isothermal pressure drop ratio of unity which cannot be achieved in realistic Brayton heat engine cycle. Further, maximum power output and thermal efficiency reflect linear variations with isothermal pressure drop ratio.

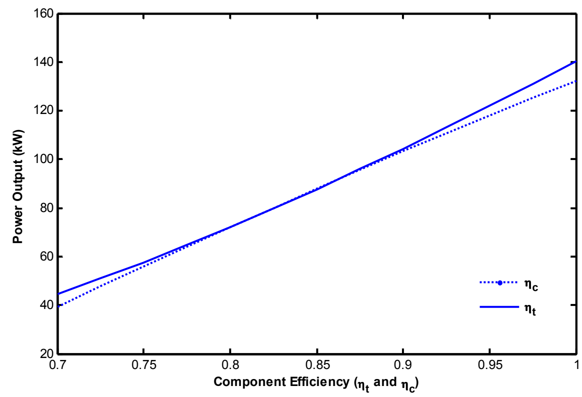

Fig. 4(a) Variations of Power Output with respect to component efficiency (η_c and η_t)

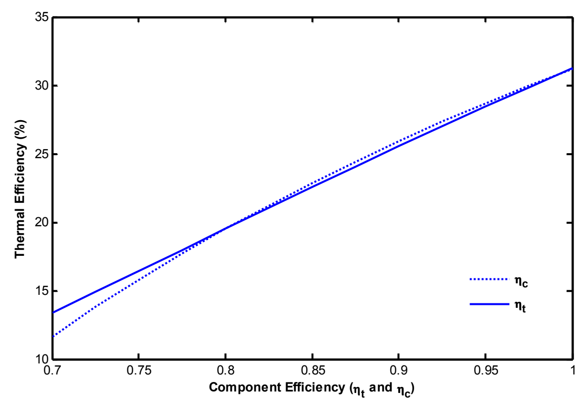

Fig. 4(b) Variations of Thermal Efficiency with respect to component efficiency (η_c and η_t)

3.5 Effects of pressure recovery coefficients ($\alpha_1 = \alpha_2$)

Fig. 6 shows the effect of pressure recovery coefficients on various performance parameters of an irreversible regenerative Brayton heat engine cycle. It is seen from these figures that maximum power output and thermal efficiency increases as the pressure drop is decreased. It is also seen from these figures that various performance parameters attains their maximum value at zero pressure drop which cannot be achieved in realistic Brayton heat engine cycle. The power output and thermal efficiency is very low at lower values of pressure recovery coefficient (α_1<0.9, α_2<0.9). Hence, efforts should be made to reduce pressure drop while designing real gas power plants.

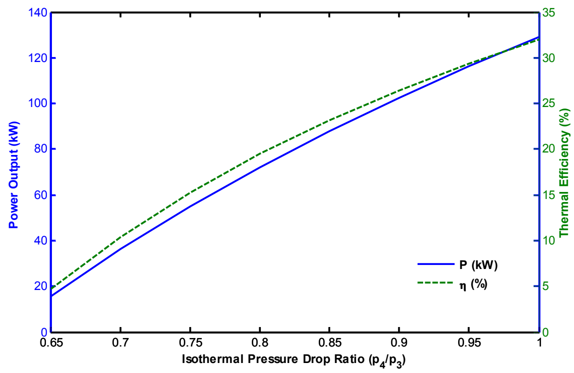

Fig. 5 Variations of power output and thermal efficiency with respect to isothermal pressure drop ratio

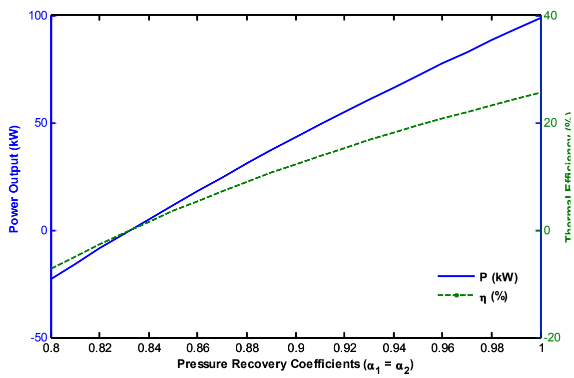

Fig. 6 Variations of power output and thermal efficiency with respect to pressure recovery coefficients

CONCLUSION

A more practical regenerative Brayton heat engine cycle model is examined in this paper. The power output is optimized in context with cycle temperature and corresponding thermal efficiency is calculated for analytical set of operating conditions. The power output and thermal efficiency is increasing with ε_{H1}, ε_L, component efficiencies, C_{H1}, C_L, pressure recovery coefficients and isothermal pressure drop ratio while its value is decreasing for ε_H, C_H. The power output is independent of regenerator side effectiveness while the corresponding thermal efficiency is increasing function of regenerator side effectiveness. It is seen that turbine efficiency adds more effect on maximum power output and the corresponding thermal efficiency as compared with compressor efficiency. It is also found that induction of two heat additions significantly enhances model efficiency above 20% as compared to conventional gas power plants. The interpolating results forms descending criterion of effectiveness of the model as ε_L, ε_{H1}, ε_H and heat capacitance rates as C_L, C_{H1}, and C_W. The above relationships are to be followed for better execution of real gas power plants. Hence, the present cycle model will be the benchmark to design and study a real cycle from thermodynamic view point.

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Nomenclature:

A= Area (m^2)
C= Heat Capacitance Rate (kWK^{-1})
k=specific heat ratio
N= Number of heat transfer units
P= Power output (kW)
Q=Heat transfer rate (kW)
T= Temperature (K)
U= Overall heat transfer Coefficient (kWm^{-2}K^{-1})
LMTD= Log Mean Temperature Difference

Greek letters:

η = Thermal efficiency
ε = Effectiveness
χ= isothermal pressure drop ratio
α= pressure recovery co-efficient

Subscripts:

H= heat source side (First heat addition)
H1=Isothermal side (Second heat addition)
L= heat sink side
R= regenerator side
s= ideal / reversible adiabatic
t = turbine
c= compressor
W= working fluid
1, 2, 3, 4, 5= state points

Appendix-I

$$X = x_1 x_3 - x_2 x_4$$
$$X_1 = x_{10}(x_5^2 - 4Xy_5)$$
$$x_1 = \varepsilon_R(1-b)(1-\eta_c)+\eta_c$$
$$x_2 = (1-a)(1-\varepsilon_R)$$
$$x_3 = \beta(1-a)(1-\eta_t^{-1})(1-\varepsilon_R)$$
$$x_4 = (1-b)\varepsilon_R$$
$$x_5 = x_1 y_3 + x_3 y_1 - x_4 y_2 - x_2 y_4$$
$$x_6 = x_1 z_3 + x_3 z_1 - x_4 z_2 - x_2 z_4$$
$$x_7 = C_W\{a(1-\varepsilon_R)+b\varepsilon_R + a_1(1-a)(1-\varepsilon_R)\}$$
$$x_8 = C_W(1-\varepsilon_R)\{a + a_1(1-a)\}$$
$$x_9 = Xy_7^2/x_7^2 - x_5 y_7/x_7$$
$$x_{10} = x_9 + y_5$$
$$Y = x_5 T_5 + x_6$$

$$Y_1 = 2x_{10}(x_5x_6 - 2Xy_6)$$
$$y_1 = (1-\varepsilon_R)(1-b)(1-\eta_c)$$
$$y_2 = (1-a)\varepsilon_R$$
$$y_3 = \beta\{\eta_t^{-1} + (1-a)(1-\eta_t^{-1})\varepsilon_R\}$$
$$y_4 = (1-\varepsilon_R)(1-b)$$
$$y_5 = y_1y_3 - y_2y_4$$
$$y_6 = y_1z_3 + z_1y_3 - z_2y_4 - y_2z_4$$
$$y_7 = C_W\{b(1-\varepsilon_R) + a\varepsilon_R + a_1(1-a)\varepsilon_R\}$$
$$y_8 = C_W\varepsilon_R\left(a_1(1-a) + a\right)$$
$$Z = y_5T_5^2 + y_6T_5 + z_5$$
$$Z_1 = x_9(x_6^2 - 4Xz_5) + y_6(x_5x_6 - Xy_6) - x_5^2z_5$$
$$z_1 = (1-\eta_c)bT_{L1}$$
$$z_2 = aT_{H1}$$
$$z_3 = \beta a(1-\eta_t^{-1})T_{H1}$$
$$z_4 = bT_{L1}$$
$$z_5 = z_1z_3 - z_2z_4$$
$$z_6 = C_W(aT_{H1} + bT_{L1} + a_1T_{H3} - aa_1T_{H1})$$
$$z_7 = C_W\left(aT_{H1}(1-a_1) + a_1T_{H3}\right)$$
$$a = \frac{C_H\varepsilon_H}{C_W}$$
$$a_1 = \frac{C_{H1}\varepsilon_{H1}}{C_W}$$
$$b = \frac{C_L\varepsilon_L}{C_W}$$