NUMERICAL INVESTIGATION OF FLOW AND HEAT TRANSFER IN COMMUNICATING CONVERGING AND DIVERGING CHANNELS

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ABSTRACT
Fluid flow and heat transfer in communicating converging and diverging channel has been numerically investigated. Channels are assumed to be at constant wall temperature and the flow is assumed to be steady state and incompressible. Since the flow and temperature fields to repeat periodically after a certain developing length, periodic boundary conditions are used for the calculations. Finite volume method is used to solve the governing differential equations numerically. Computations are performed for different values of the plate angles and Reynolds numbers. Moreover, velocity distributions along the flow field are illustrated. It was found that the converging-diverging channels destroy the boundary layer significantly and Nusselt number is found to be about 400% higher than those of parallel plate channels, whereas due to vortex formation, pressure drop increases also.

Keywords: Communicating Converging and Diverging Channel, Numerical, Periodic Boundary Condition, Boundary Layer

INTRODUCTION
It is very important to use compact heat exchangers in many engineering branches such as solar collectors, heaters, refrigerators, automotive and electronics equipment. Especially for the design of compact heat exchangers heat transfer must be maximized. Therefore, this area is investigated by many researchers [1-4].

There are many forms of heat transfer enhancements. Disrupting the growth of the boundary layer and mixing are one of the most applied methods for enhancement of heat transfer between the flowing fluid and heat transferring surfaces. For this application interrupted-wall channels were investigated, and it was showed that heat transfer can be increased using this method [5-8].

Converging and diverging channels are used in industry especially for increasing efficiency of heat exchangers. With these channels heat transfer surface area per volume is increased, boundary layers are disrupted and specially mixing regions are created to improve heat transfer coefficient. For this issue, there have been a lot of studies by researchers. Mendes and Sparrow [9] performed experimental study for the converging and diverging tubes to determine the entry and fully developed region heat transfer coefficients, pressure distribution and friction factor at various taper angles. Garg and Maji [10] numerically demonstrated that the converging and diverging channel configuration is an effective technique for enhancing of heat transfer.


Flow and heat transfer in periodic sinusoidal channels are investigated by Wang and Vanka [13] They showed that using these channels heat transfer can be enhanced with appropriate pressure drop.

Kotcioglu and Ayhan [14] accomplished evaluation of performance parameters in the turbulent regime regarding the enhancement of the heat transfer using winglet-type vortex generators. The heat transfer characteristics and flow structure in turbulent flow in a rectangular channel containing built-in wing type vortex generator, have been investigated experimentally. The vortices in this flow caused enhanced heat transfer. Dejond and Jacobi [15] investigated experimentally mass transfer at interrupted plate arrays and found that mass transfer is higher than theoretically calculated values for these systems. Converging and diverging communicating form of channel are used as finned surfaces by Caliskan and Baskaya [16] and Kotcioglu et al.[17]. Chunhua Min et al.[18] presented modified rectangular longitudinal vortex generator obtained by cutting off the four corners of a rectangular wing experimentally. Chunhua Min et al. [19] studied numerically comparative investigation longitudinal vortex generators for turbulent flow. Moreover, Gholami et al. [20] investigated winglet type vortex
generators in fin and tube compact heat exchanger and found that vortex generators enhance thermal mixing of the fluid and delay the boundary layer separation on the tubes.

Heat transfer in communicating converging and diverging channels are first investigated theoretically and experimentally by Yılmaz and Ayhan [21]. They have showed that due to mixture of the flow perpendicular to flow direction, heat transfer increases %160 compared to parallel plate channels. They supplied experimental results which show that heat transfer enhancement increases with increasing Reynolds number. There are lots of studies about implementation of periodic boundary condition in fluid flow and heat transfer analysis [22-24].

The communicating converging-diverging channels can be used in automobile radiators, PV collectors, gas-gas heat exchangers, liquid-liquid plate heat exchangers, etc. Numerical investigation of communicating converging channel is introduced by Yılmaz and Erdinç [25]. To the best of authors’ knowledge there is not any other numerical investigation in the open literature concerning the communicating converging-diverging channels. In this study, detail numerical investigation is carried out on enhancement of forced convection heat transfer using communicating converging-diverging channels under the assumptions of two dimensional, viscous, incompressible, fully developed and laminar flow condition. For the validation of calculations, parallel plate channel is considered and obtained results were compared with the well-known values available in the literature for the convective heat transfer in channels.

**MODEL DESCRIPTION**

Two-dimensional converging-diverging and communicating channel is shown in Figure 1. After a short entrance region, a periodic fully developed region is demonstrated in figure 2. It is assumed that the fluid is incompressible and laminar, constant wall temperature and steady-state conditions. The fluid properties are assumed to be constant.

![Figure 1. Converging-Diverging channel and computational domain A](image)

For two-dimensional incompressible flow continuity and momentum equations for x and y direction can be written as follows [25]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]
\[
\begin{align*}
\frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial P}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \\
(3)
\end{align*}
\]

For two dimensional, steady, incompressible and laminar flow energy equation is written as:

\[
\begin{align*}
\frac{u}{\partial x} + v \frac{\partial T}{\partial y} &= a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \\
(4)
\end{align*}
\]

Here \( a \) is thermal diffusivity.

The associated boundary conditions are demonstrated for hydrodynamically fully developed flow and are presented in figure 2 and these are given below:

\[
\begin{align*}
u_A &= u_B \\
v_A &= v_B \\
(5)
\end{align*}
\]

\[
\begin{align*}
\Delta P_p &= \text{pressure loss in one periodic element of the length } L \text{ in the channel.}
\end{align*}
\]

\[
\begin{align*}
\frac{T(y,x) - T_{\text{wall}}}{\bar{T}(x) - T_{\text{wall}}} = \frac{T(y,x+L) - T_{\text{wall}}}{\bar{T}(x+L) - T_{\text{wall}}} \\
(8)
\end{align*}
\]

where \( \bar{T}(x) \) and \( \bar{T}(x+L) \) are mean temperatures at \( x \) and \( x+L \). Mean temperature is defined as follows:

\[
\bar{T}(x) = \frac{\int_{0}^{R(x)} uTydy}{\int_{0}^{R(x)} uyd} \\
(9)
\]

\( R(x) \) is shown in figure 1. For symmetry boundary conditions below equation can be written:

\[
\begin{align*}
\frac{\partial u(x,y)}{\partial y} &= 0, \quad v(x,y) = 0, \quad \frac{\partial T(x,y)}{\partial y} = 0
\end{align*}
\]

Reynolds number and equivalent diameter are defined as follows:
\[ Re = \frac{\rho u_m d_e}{\mu} \] (11)

\[ d_e = \frac{4 \cdot V}{F_p} = \frac{(H \cdot L)}{(L_p + t)} \] (12)

where \( H \) is the width of the channel and \( u_m \) is mean velocity of air at the inlet of the computational domain.

**SOLUTION**

The steady, laminar and incompressible Navier-Stokes equations were discretized using the finite volume method for flow domain demonstrated in figure 1 and 3. ANSYS FLUENT computer program was used to solve the governing equations. QUICK scheme was applied for convective and diffusive terms. The pressure–velocity coupling algorithm SIMPLE was selected. Meshes were implemented to the flow domain as illustrated in figure 4. The check of mesh independence is necessarily to ensure the accuracy and validity of the numerical results. So, the mesh independence study is conducted on the all channels. The four parameters of interest for the present case are friction factor, Nusselt number, mixing ratio and thermal enhancement factor. Convergence criteria for momentum and continuity equations are \( 10^{-6} \) and convergence criteria for energy equations are \( 10^{-9} \).

\[ Nu_x = \frac{h(x)d_e}{k} \] (13)

where \( h(x) \) and \( k \) are local heat transfer coefficient and thermal conductivity of air respectively. \( h(x) \) is defined as follows:
\[ h(x) = \frac{\dot{q}_x}{T_w - \bar{T}(x)} \]  

(14)

\( \dot{q}_x, \bar{T}(x) \) are heat flux at the point \( x \) and mean temperature.

Mean heat transfer coefficient and Nusselt number are determined as follows:

\[ h = \frac{\dot{Q}}{F_p \Delta T_m} \]  

(15)

where \( F_p \) is plate surface area:

\[ F_p = 4 \cdot (L_p + t) \cdot B \]  

(16)

and \( B \) is the width of the channel.

\( \dot{Q} \) is determined from the equation

\[ \dot{Q} = \dot{M} \cdot c_p \cdot (T_e - T_i) = h \cdot F_p \cdot \Delta T_m \]  

(17)

where \( \dot{M}, c_p, T_i, T_e \) are mass flow rate, specific heat and fluid inlet/exit temperature at the inlet and exit of the computational domain. \( \Delta T_m \) is logarithmic mean temperature difference:

\[ \Delta T_m = \frac{(T_w - T_i) - (T_w - T_e)}{\ln \left( \frac{T_w - T_i}{T_w - T_e} \right)} \]  

(18)

Mean Nusselt numbers are calculated then as:

\[ Nu = \frac{h d_e}{k} \]  

(19)

Friction factor is defined as usual:

\[ f = \frac{\Delta P_p d_e / L}{\rho \frac{u_m^2}{2}} \]  

(20)

where \( \rho \) is density of the fluid.

Thermal enhancement factor is defined as follows [26]:

\[ \eta = \frac{Nu}{Nu_p} \left( \frac{f}{f_p} \right)^{1/3} \]  

(21)

\( Nu, f \) and \( Nu_p, f_p \) are Nusselt number and friction factors for the calculated channels and parallel plate channels respectively.

For the numerical solution temperature and geometric parameters are given in table 1 and 2.
Table 1. Boundary conditions

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_w$(Wall)</td>
<td>363.15 K</td>
</tr>
<tr>
<td>$T_{air}$(inlet)</td>
<td>303.15 K</td>
</tr>
</tbody>
</table>

Table 2. Geometric dimensions of flow domain.

<table>
<thead>
<tr>
<th>Geometric parameter</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>40 mm</td>
</tr>
<tr>
<td>t</td>
<td>2 mm</td>
</tr>
<tr>
<td>e</td>
<td>10 mm</td>
</tr>
<tr>
<td>H</td>
<td>25 mm</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0-30°</td>
</tr>
</tbody>
</table>

RESULTS AND DISCUSSIONS

Since there is not any study to compare with this system, validation of the heat transfer and friction factor of the parallel plate is performed by comparing with the previous values under similar operating conditions for hydrodynamically and thermally fully developed flow.

As seen in figure 5 and 6, the present numerical results are found to be in good agreement with exact solution values obtained from the literature [27] for both the Nusselt number and the friction factor. Also flow in interrupted wall channel without angle is compared with Yilmaz [5] using same parameters. Yilmaz [5]’s results are shown in figure 7. Numerical results are again found to be in good agreement with the results obtained by Yilmaz [5] as seen in figure 8 for $L^*=40$. This shows a strong confidence in further investigation of the communicating converging diverging channels.

Figure 5. Comparison of friction factor between the values calculated in this work and analytical results for parallel plate channel.
Figure 6. Comparison of Nusselt number between the values calculated in this work and analytical results for parallel plate channel.

Figure 7. Arrangement of plates in Yilmaz [5]’s work.
Figure 8. Comparison of friction factor between the values calculated in this work and Yilmaz [5]’s work for $L^* = 40$ ($L^* = L/2s$)

It is expected that there exists a certain optimum angle for these channels. Therefore, especially this optimum value of the angle should be determined. For a given Reynolds number (Re=100), the mixing effect of the angle ($\alpha$) is investigated. Flow planes for the explanation of the flow are shown in figure 9.

![Flow areas](image)

Figure 9. Flow areas

To show the mixing effect of the flow between the channels a mixing ratio is defined:

$$\tilde{M}^* = \frac{\dot{M}_b}{\dot{M}_a}$$

(22)

$\dot{M}_b$ and $\dot{M}_a$ are mass flow rate of fluid passing through area b and a. Calculated mixing ratios are given in table 3 and shown in figure 10. As seen from the figure 10, there is a maximum mixing point at $\alpha = 20^\circ$. At the maximum point, 26% of the mass flow is transferred to the neighboring channel.

Figures 11, 12, 13 and 14 respectively show velocity distributions and streamlines for Re=100. The figures indicate that communicating converging and diverging channels not only can interrupt thermal boundary layer, but also enhance mixing of the fluid. This is due to pressure and velocity differences across the passages between converging-diverging channels. For different angles $\alpha$, mixing of the flow in adjacent channels are clearly seen in these figures. However, vortex formation occurs after certain plate angle of 15°, and this causes rapid increase of pressure drop.
Table 3. Effect of angle ($\alpha$) on mixing ratio

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$M^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.1055</td>
</tr>
<tr>
<td>10</td>
<td>0.1916</td>
</tr>
<tr>
<td>15</td>
<td>0.2479</td>
</tr>
<tr>
<td>20</td>
<td>0.2664</td>
</tr>
<tr>
<td>25</td>
<td>0.2569</td>
</tr>
<tr>
<td>30</td>
<td>0.2382</td>
</tr>
</tbody>
</table>

Figure 10. Effect of angle ($\alpha$) on mixing ratio

Figure 17 shows the thermal enhancement factor for such channels for different plate angle in the Re range of 100-600. As outlined above, due to vortex formation thermal enhancement factor is greater than 1 for $\alpha \leq 15$. For $\alpha \geq 15$, as Reynolds number increases, thermal enhancement factor increase also. The thermal performance factor above unity indicated that the effect of heat transfer enhancement due to the enhancing device was more dominant than the effect of rising friction and vice versa [28].

Effect of plate angle on friction factor and Nusselt number for different Reynold numbers are shown in figure 15 and 16. As plate angle increases Nusselt number increases because of mixing effect and vortex formation. However, friction factor increases due to intensive vortex formation with the increase of plate angle. It can be seen also that when Re number increases; Nusselt number increases, and friction factor decreases as expected.
**Figure 11.** Velocity vectors and streamlines for $\alpha = 10^\circ$

**Figure 12.** Velocity vectors and streamlines for $\alpha = 15^\circ$
Figure 13. Velocity vectors and streamlines for $\alpha = 20^\circ$

Figure 14. Velocity vectors for $\alpha = 30^\circ$
Figure 15. Variation of friction factor with plate angle for different Reynolds number

Figure 16. Variation of Nusselt number with plate angle for different Reynolds number
CONCLUDING REMARKS

Two-dimensional numerical investigation of flow and heat transfer in communicating converging-diverging channel has been performed under assumptions of viscous, incompressible fully developed and laminar flow condition. Especially mixing of the fluid in adjacent channels are determined and illustrated. The following conclusions can be drawn:

- Due to pressure and velocity differences across the passages between converging-diverging channels, mixing effect between channels is obtained.
- The converging-diverging channels mix the fluid perpendicular to the flow directions and destroy the boundary layer significantly and thereby increase the heat transfer coefficient to develop a more effective heat exchanger.
- It is shown that at Re=100 maximum mixing occurs at the angle $\alpha = 20^\circ$.
- Nusselt number and friction factor are increasing with the increasing of angle $\alpha$ and Reynolds number.
- Thermal enhancement factor is higher than 1 for $\alpha < 15^\circ$.
- These channels can be used in automobile radiators, PV collectors, gas-gas heat exchangers, liquid-liquid plate heat exchangers and etc.

ACKNOWLEDGEMENTS

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NOMENCLATURE

- $c_p$: specific heat [J/kg]
- $d_e$: equivalent diameter[m]
- $f$: friction factor, eq.(20)
- $F_p$: heat transfer surface area[m$^2$]
- $h$: heat transfer coefficient[W/m$^2$.K]
REFERENCES


