



### Research Article

## A NEW METHOD FOR THE SIZE AND PERFORMANCE ANALYSES AND OPTIMIZATION OF THERMAL SYSTEMS: THE EXERGY DENSITY

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### ABSTRACT

With the acceptance of thermodynamics as a science field, an important step has been taken in the transition from thermal systems produced by trial and error method to planned and calculated systems which are produced according to determined standards. During this time, many performance criteria have been proposed and some of them have been widely used in design phases and in practical applications. In this study, the interaction between the performance and size of the thermal systems is examined by using a new proposed criterion, the exergy density, as a thermodynamic state property. The effects of pressure and temperature of working fluid on the exergy density values are obtained and property graphics are plotted. In addition to these, case studies for the air standard cycles (Otto, Diesel, Atkinson and Brayton) and a steam power (Rankine) cycle have been carried out. As a result, the proposed criterion or property for thermal systems has been shown to be beneficial in the design phase as it examines system dimensions as well as system performance.

**Keywords:** Exergy, exergy density, thermal systems, exergetic size.

### NOMENCLATURE

E	Energy (kJ)
ex	Specific exergy (kJ/kg)
h	Specific enthalpy (kJ/kg)
m	Mass (kg)
$\rho$	Density (kg/m <sup>3</sup> )
$\rho_{ex}$	Exergy density (kJ/m <sup>3</sup> )
P	Pressure (kPa)
q	Heat transfer (kJ/kg)
s	Specific entropy (kJ/kg.K)
T	Temperature (K)
u	Specific internal energy (kJ/kg)
U	Energy density (kJ/kg)
w	Specific work (kJ/kg)
v	Specific volume (kg/m <sup>3</sup> )
V	Volume (m <sup>3</sup> )
VC	Volume coefficient (kJ/m <sup>3</sup> )

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VFR Volumetric flow ratio

### Subscripts

- a Atkinson cycle
- b Brayton cycle
- d Diesel cycle
- i State point
- ic Isentropic efficiency of compression
- ie Isentropic efficiency of expansion
- in Inlet
- max Maximum
- min Minimum
- o Otto cycle
- out Outlet

## 1. INTRODUCTION

Thermodynamics began to be accepted as a modern science after Carnot’s study “*Reflections on the Motive Power of Fire*” which is related to heat, power and their conversion. Carnot’s approach was a milestone and success for thermodynamics and this achievement has been indicated by Sandler and Woodcock [1] with these words “*Early engines were designed empirically without the knowledge of thermodynamics, but their performance encouraged leading engineers to become scientists.*” From the 1800s to 2000s many definitions have been made by scientist or engineers with different perspectives and approaches to give information about system performance [2]–[6].

We will not be able to access realistic information about the dimensions of the component or system that we want to design if our working capacity or availability alone is a criterion. In order to be able to overcome this lack of definition in the literature, in this study, it will be examined the exergy and size relation as a recently offered independent function (the exergy density) and then we will show what this function physically expresses and the effects of pressure and temperature changes on the exergy density of frequently used fluids in thermal systems. In addition to this, to prove that the exergy density is a thermodynamic property, the most common used thermodynamic power cycles built up using the exergy density.

## 2. THERMODYNAMIC MODEL

The concept of exergy density could be useful for the performance and size analysis of thermal systems including steam power plants, especially for volumetric fluid machines. Prior to defining the exergy density, we need to explain the other concepts that are relevant to our new definition in order to get a better understanding. In this context, the exergy density will be defined after describing the specific volume, density, specific enthalpy & energy conservation, specific entropy & second law, exergy, energy density, volumetric flow rate and volume coefficient.

*The specific volume* gives information about a relationship between volume and mass and defines the material or fluid size which is shown in Eq. 1 as  $m^3/kg$ . Beside this, *the density* can be expressed as the ratio of unit mass to the unit volume and is also equal to the inverse of the specific volume. Its unit is  $kg/m^3$  that is shown as in Eq. 2.

$$v = V / m \tag{1}$$

$$\rho = m / V = 1 / v \tag{2}$$

where  $v$  refers to specific volume,  $\rho$  is the density,  $m$  is the mass and  $V$  refers to volume.

*The specific enthalpy*,  $h$ , is a property of a substance, like temperature, specific volume etc. and is defined as the sum of specific internal energy and boundary work [1], [7], [8]. It can be

defined by Eq. 3 in kJ/kg unit but can't be measured directly so a reference point is necessary for calculation and is related to *the first law of thermodynamic*, Eq. 4, which deals with the conservation of energy and its conversion [9].

$$h = u + P\Delta v \tag{3}$$

$$\Delta u = \Delta q - \Delta w \tag{4}$$

where  $h$  refers to specific enthalpy,  $u$  is the specific internal energy,  $P$  is the pressure,  $\Delta v$  is the difference between specific volumes,  $\Delta u$  is the difference between specific internal energies,  $\Delta q$  is the transferred heat and  $\Delta w$  refers to the specific work.

*The law of entropy* which was given by Clausius as an output of Eq. 5 is a criterion about the irreversibilities of the system in kJ/kg.K unit [1], [8], [10] that was also called *the second law of thermodynamics* [9].

$$\oint \frac{\delta q}{T} \leq 0 \tag{5}$$

where  $q$  refers to heat transfer and  $T$  refers to source temperature.

The exergy refers to the availability or quality of a thermodynamic system with respect to a specified reference [11] and is related to the first and second laws of thermodynamic. The availability of a thermal system is zero when in balance with the reference conditions [9]. The physical exergy definition is given in Eq. 6 where  $h_i$  and  $h_0$  refer to the specific enthalpies and  $s_i$  and  $s_0$  refer to the specific entropies of  $i^{th}$  point and dead state conditions and  $T_0$  is the dead state or ambient temperature.

$$ex_i = h_i - h_0 - T_0 \cdot (s_i - s_0) \tag{6}$$

*The energy density (U)* is the quantity of energy stored (E) in a given system or region of space per unit volume (V) in J/m<sup>3</sup> unit [12] as in Eq. 7 and is generally used as a useful criterion for fuels.

$$U = \frac{E}{V} \tag{7}$$

*Volumetric flow ratio (VFR)* is a dimensionless parameters especially for expanders and pumps that can be described the ratio of volumetric flow rates of the working fluid at the outlet and inlet sections, Eq. 8.

Another volumetric criteria is, which relates to VFR, *Volume coefficient (VC)* is also described in Eq. 9 as the ratio of the specific volume at outlet section for expanders or inlet section for pumps to the theoretic enthalpy drop of expander or pump [13] in kJ/ m<sup>3</sup> unit.

$$VFR = \frac{\dot{V}_{out}}{\dot{V}_{in}} \tag{8}$$

$$VC = \frac{v_{out/in}}{\Delta h} \tag{9}$$

The definitions above do not give information about both the size and real performance of the system, which does not allow us to make a real assessment. For example, the specific volume of a fluid with a high exergy value is so large that the construction of a system will cause the dimensions to grow quite large, which will lead to increased costs. On the other hand, the exergy density of a fluid with a low exergy can be high. Beside this, it would be easier and more economical to design and manufacture a system with a working range of fluid with a higher exergy value and lower specific volume. Therefore, by defining a new state property, *the exergy*

density ( $\rho_{Ex}$ ) [14], a new dimension is added to the concept of the exergy, and by explaining the relevance of the exergy to dimensions, more realistic cost calculations and appropriate approaches for optimizations will be developed for thermal systems. The expression of exergy density is defined in Eq. 10 as the ratio of the physical exergy to the specific volume at that point by  $\text{kJ}/\text{m}^3$  unit.

$$\rho_{Ex,i} = \frac{ex_i}{v_i} \tag{10}$$

where  $ex$  refers to specific exergy and  $v$  refers to specific volume.

In this study, the exergy density, which is called an intensive property of the system and also is a point function so that no matter which process path is followed, has been applied to water and air used thermal systems by using Engineering Equation Solver (EES) software [15].

### 3. RESULTS AND DISCUSSION

The result of exergy density analyses were given in figures 1-4 for water and in figures 5-8 for air in terms of different pressure and temperature conditions. The interaction between specific enthalpy, specific exergy, exergy density and specific entropy variations are given in Figure 1 from 10 kPa to 22000 kPa for saturated liquid and saturated vapor of water. The dead state or ambient conditions for water are 8 kPa and 298 K. It can be seen from the figure that the specific enthalpy and specific exergy have a maximum value with respect to specific entropy in the saturated vapor state but the exergy density has a maximum value in the saturated liquid state so it can be optimized for working conditions or purpose of using.

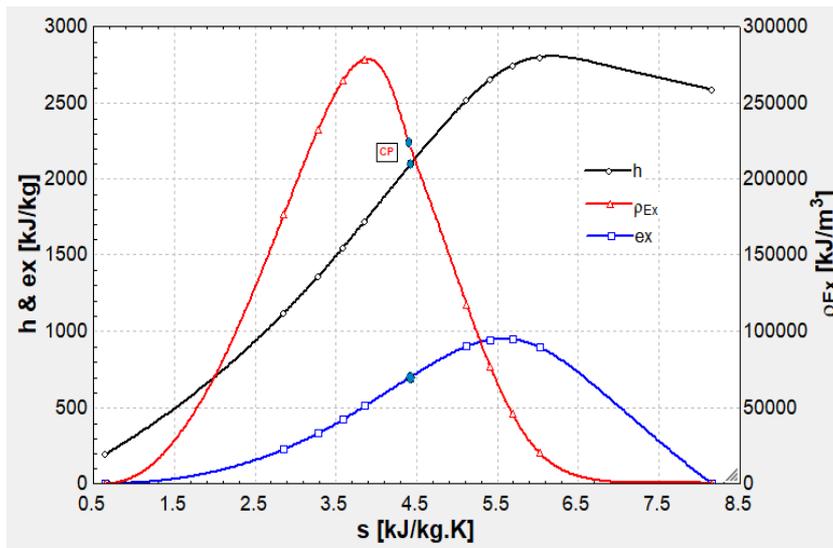


Figure 1. Specific enthalpy, specific exergy, exergy density and specific entropy diagram for water

The most important purpose of the paper is to use exergy density for the performance and size analysis of thermal systems. Therefore it should be very important to understand the exergy density-specific entropy diagram for various pressures, qualities, and temperatures. In order to make the previous sentence clear, we have to use both specific exergy-specific entropy and

exergy density- specific entropy diagrams for the same conditions at Fig. 2 and 3. For example, although chosen two points (the first is 15 MPa & 800 K and the second is 1 MPa & 1000 K) that have equal specific exergy values in Fig. 2, the exergy density value of the first point is nearly 20000 times of the second point as seen in Fig. 3.

The detailed specific exergy - specific entropy diagram is shown in Fig. 2 and the exergy density- specific entropy diagram is shown in Figure 3 for different pressures, temperatures and qualities of steam with saturated liquid and saturated vapor curvatures at a constant ambient temperature and pressure values. It gives us some important design criterions for steam power plants which can be used for performance and size calculations.

The black lines on both sides of the critical point (CP) are shown the saturated liquid and saturated vapor of the specific exergy & exergy density correspond to specific entropy for different pressure. Beside this, the blue dotted lines for different quality values, the red straight lines for different pressure values from 1 MPa to 20 MPa and the blue straight lines for the different temperature values between 600-1000 K give us the specific exergy & exergy density-specific entropy interaction. This diagram can be plotted for different working fluids that use for heat engines, refrigeration machines and heat pumps and will provide some beneficial information to designers for the optimization of thermal systems' performance and dimension.

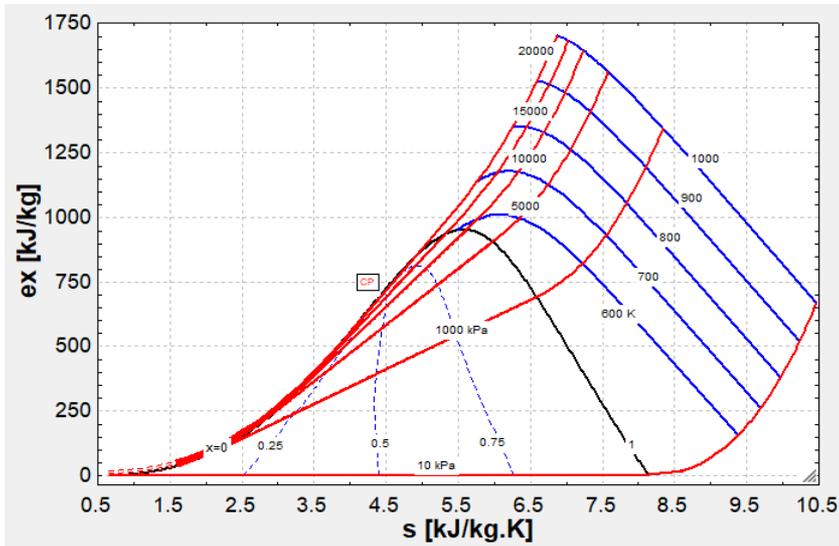


Figure 2. Specific exergy and specific entropy diagram for water

The case power plant is located in the Southern Marmara region of Turkey and has 320 MW installed power by 40% thermal efficiency. The flow diagram and more technical specifications about the plant can be found in [16], [17]. The application of the exergy density will be implemented for a steam power plant operating at turbine section (1-2, 3-4 and 4-5) levels, reheating (2-3), condensing (5-6), pumping (6-7) and boiler section (7-x-y-1). Physical properties (P, T, v, h, s, ex and  $\rho_{Ex}$ ) of state points (i) of the simplified (steam extractions to the feed water heaters are neglected) case power plant are given in Table 1.

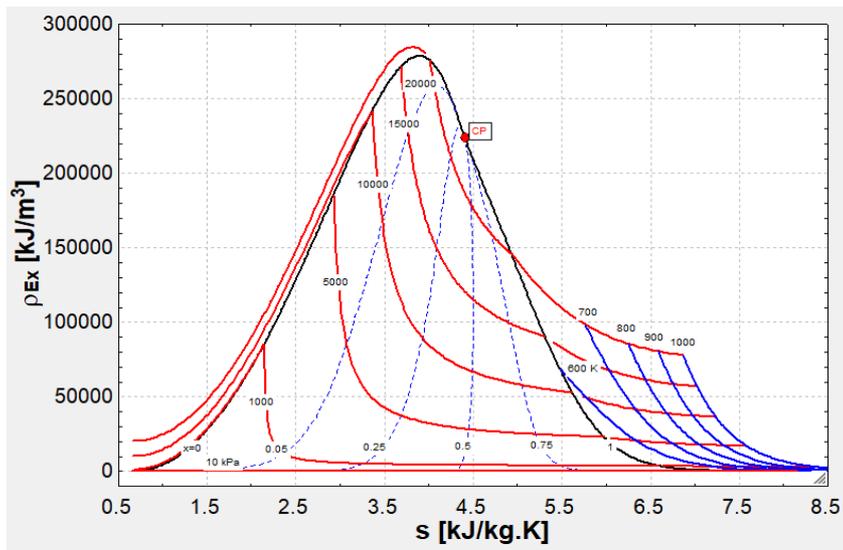
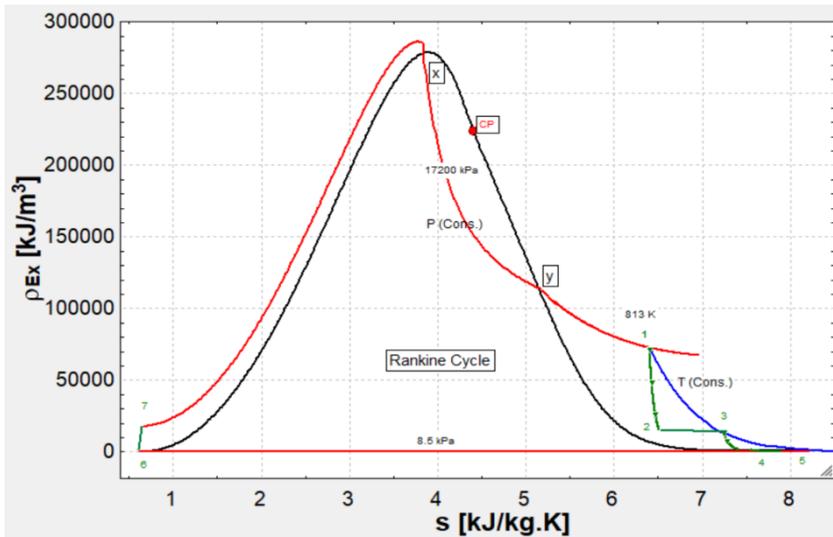


Figure 3. Exergy density and specific entropy diagram for water

Table 1. Physical properties of state points of the case power plant (Simplified)

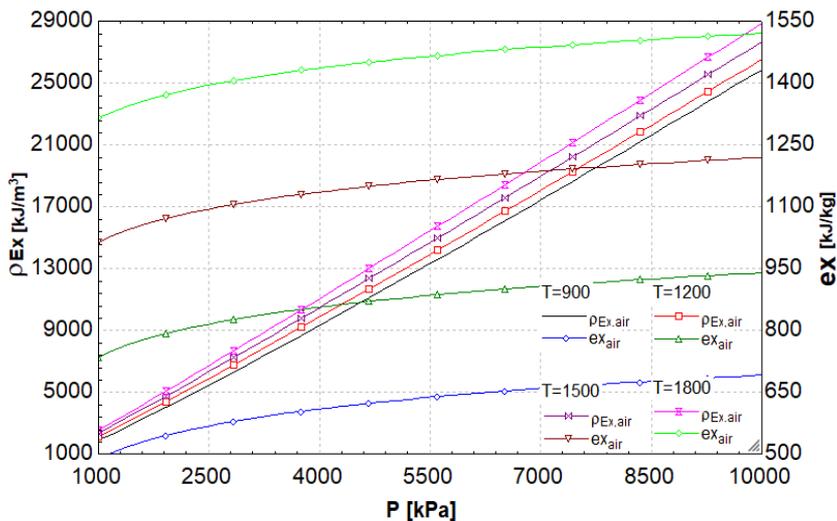
	P	T	v	h	s	ex	ρ <sub>Ex</sub>
i	bar	K	m <sup>3</sup> /kg	kJ/kg	kJ/kg.K	kJ/kg	kJ/m <sup>3</sup>
0	0.08	298	0.001008	173.8	0.5925	0	---
1	172	813	0.01931	3398	6.403	1395	72352
2	40	685	0.0602	3242	6.812	981	16320
3	37	813	0.09905	3540	7.246	1273	12851
4	5.2	600	0.4726	3120	7.537	689	1517
5	0.085	315	15	2476	7.756	7.8	0.5
6	0.085	315	0.001009	178.7	0.6078	0.00934	9.256
7	172	320	0.001003	211.1	0.6550	17.46	17402
x	172	626	0.00178	1699	3.821	508.9	285405
y	172	626	0.008196	2541	5.165	928	113229

The  $\rho_{Ex}$ - $s$  diagram of the case power plant (a Rankine cycle) is given in Fig. 4 with assuming the isentropic efficiencies of pump and turbine are 0.85 and the pressure losses are neglected in the heat transfer units. The produced power (kW) and power production per unit volume (kW/m<sup>3</sup>) can be obtained via using steam flow rate at calculations. It can be seen from the Fig. 2 and Fig. 4 which turbine is more efficient on behalf of the system performance and size that exergy drop in high pressure turbine is smaller than the low pressure turbine, but the exergy density changing in high-pressure turbine is larger than the low pressure. In addition, the reheat leads to increased exergy but decreased exergy-density. In this context, it is very important to prevent high-pressure turbine loses. Thus, by this property, we can make the right decision for our purpose of using.



**Figure 4.** Exergy density and specific entropy diagram for the case thermal power plant

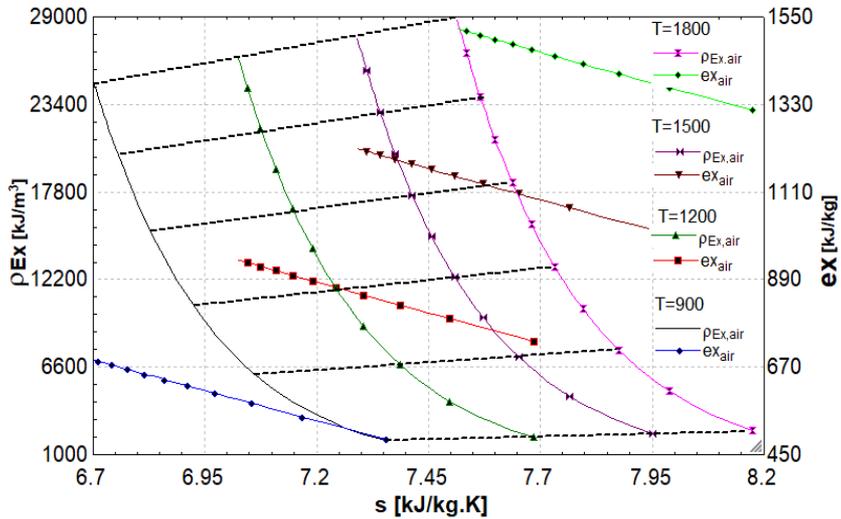
The effect of temperature and pressure changing on exergy density and exergy values of the air are given in Figure 5 for 1000-10000 kPa pressure and 900-1800 K temperature values. The exergy density value of air is highly increases with pressure increasing but exergy is less increasing. Beside this, the temperature increases the exergy more than pressure.



**Figure 5.** The effects of pressure and temperature on the specific exergy and exergy density of air

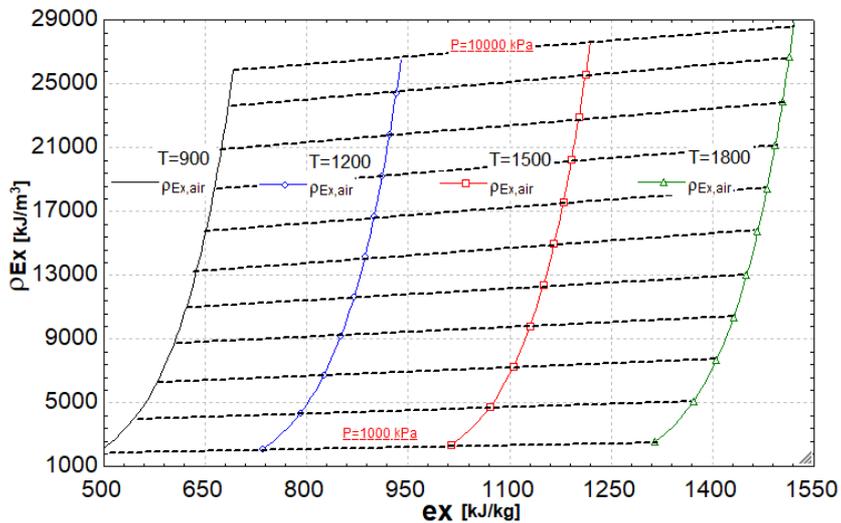
The exergy, exergy density and specific entropy variations via pressure and temperature changing are also given in Figure 6. It is clearly seen from the figure that although the exergy and specific entropy variation has linear characteristic, the exergy density has an exponential

curvature and the maximum points of exergy and exergy density at temperature curves are for maximum pressure.



**Figure 6.** The effects of pressure and temperature on the specific exergy, exergy density and specific entropy of air

The exergy density and exergy interaction via pressure (1000 kPa-10000 kPa) and temperature (900-1800 K) variations are given in Figure 7 that gives some information about which selection is appropriate for the designed system, single large system or multiple small system?



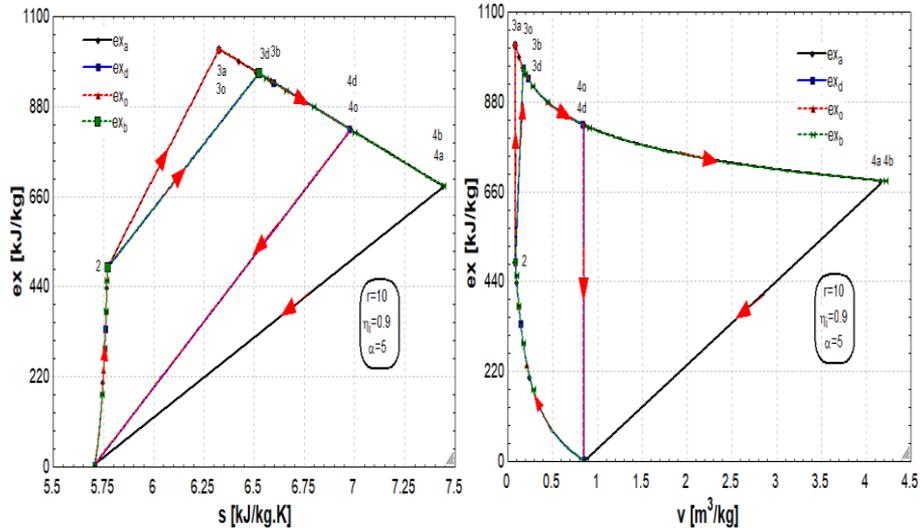
**Figure 7.** The effects of pressure and temperature on the specific exergy and exergy density of air

Thermodynamic formulations of state properties ( $T, P, v, h, ex$ ) of each cycle points can be calculated as a function of volumetric compression ratio ( $r=v_1/v_2$ ), the ideal gas constant ( $k$ ), the ratio of the maximum temperature to the minimum ( $\alpha=T_{max}/T_{min}$ ), isentropic efficiencies of compression and expansion processes, ( $\eta_{ic}, \eta_{ie}$ ) and inlet conditions of cycle ( $T_1, P_1$  and  $v_1$ ) with using ideal gas law formulations. Magnitudes of the physical specifications of state points are given in Table 2. The values of the points 1 and 2 are equal for all cycles but the points 3 and 4 are different. The analyses show that Atkinson and Otto cycles have the maximum values for the point 3 and also Atkinson and Otto cycles have the minimum values for the point 4.

**Table 2.** State points magnitude of the air power cycles

i	ex <sub>a</sub>	ex <sub>o</sub>	ex <sub>d</sub>	ex <sub>b</sub>	ρ <sub>Ex,a</sub>	ρ <sub>Ex,o</sub>	ρ <sub>Ex,d</sub>	ρ <sub>Ex,b</sub>
	kJ/kg				kJ/m <sup>3</sup>			
1	0	0	0	0	0	0	0	0
2	487	487	487	487	5656	5656	5656	5656
3	1020	1020	960.9	961	11849	11849	5607	5607
4	50.71	161.1	289.3	101.9	36.02	187.1	336	59.45

The specific exergy and exergy density diagrams with specific entropy and specific volume for the most commonly used air standard heat engine cycles such as Otto (o), Diesel (d), Atkinson (a) and Brayton (b) are given in Figure 8 and 9 which are modeled as four processes (compression and expansion with the efficiency of 0.90, heat addition and heat rejection at constant volume and/or constant pressure) in EES. It is also assumed that the pressure drops are neglected, the temperature ratio is 5, volumetric compression ratio is 10, specific heat capacities are constant for all temperature values and dead state conditions are 300 K and 100 kPa for case study analysis.



**Figure 8.** Specific exergy diagrams via the specific entropy and specific volume for air standard power cycles

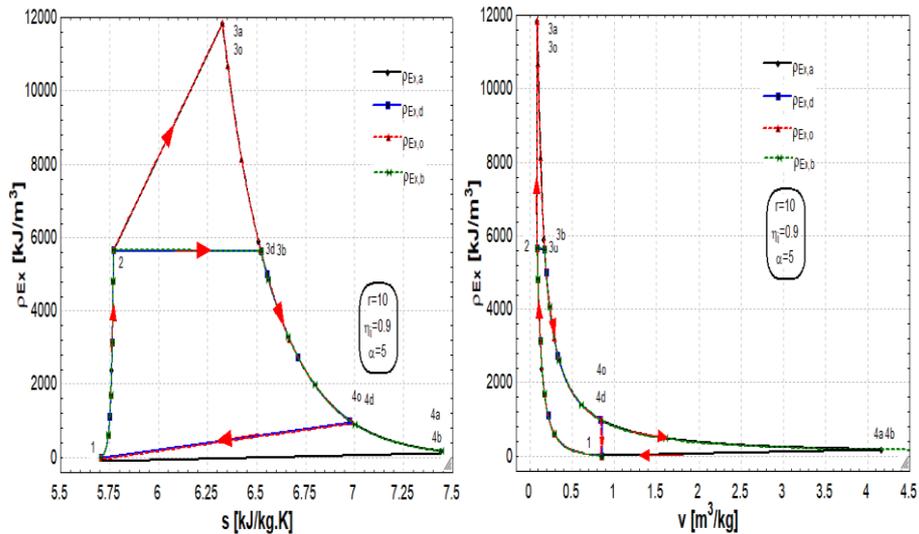


Figure 9. The exergy density diagrams via the specific entropy and specific volume for air standard power cycles

#### 4. CONCLUSIONS

Parametric and comparative analyses of the new offered exergetic size definition, the exergy density, have been done for water and air fluids that are widely used in thermal systems such as heat engines, refrigeration machines and heat pumps. Detailed specific exergy - specific entropy and the exergy density - specific entropy diagrams are obtained for different pressure, temperature and steam quality values just like specific enthalpy - specific entropy diagram (Mollier diagram) and also same results and diagrams were obtained of air for different temperature and pressure. In order to show the exergy density is a thermodynamic property, a steam power cycle (Rankine) in the exergy density - specific entropy axes and four air standard power cycles (Diesel, Otto, Atkinson, and Brayton) were generated in the exergy density - specific entropy & specific volume axes. As a result of the analyses, it can be seen that the exergy density can be applied to the thermal cycles and it will become a more realistic approach and a new criterion for designers in the design phase.

#### REFERENCES

- [1] R. Battino, S. E. Wood, and L. E. Strong, "A Brief History of Thermodynamics Notation," *J. Chem. Educ.*, vol. 74, no. 3, p. 304, Mar. 1997.
- [2] S. Carnot, *Réflexions sur la puissance motrice du feu et sur les machines propres à développer cette puissance*. Bachelier Libraire, 1824.
- [3] W. J. M. Rankine, *A Manual of the Steam Engine and Other Prime Movers*. R. Griffin, 1859.
- [4] R. Mollier, *Neue Diagramme zur Technischen Wärmelehre*. 1904.
- [5] I. I. Novikov, "The efficiency of atomic power stations (a review)," *J. Nucl. Energy* 1954, vol. 7, no. 1-2, pp. 125-128, Aug. 1958.
- [6] F. L. Curzon and B. Ahlborn, "Efficiency of a Carnot engine at maximum power output," *Am. J. Phys.*, vol. 43, no. 1, pp. 22-24, Jan. 1975.

- [7] I. K. Howard, "H Is for Enthalpy, Thanks to Heike Kamerlingh Onnes and Alfred W. Porter," *J. Chem. Educ.*, vol. 79, no. 6, p. 697, Jun. 2002.
- [8] S. I. Sandler and L. V. Woodcock, "Historical Observations on Laws of Thermodynamics," *J. Chem. Eng. Data*, vol. 55, no. 10, pp. 4485–4490, Oct. 2010.
- [9] Y. A. Çengel and M. A. Boles, *Thermodynamics: an engineering approach, 8th Edition*, 8th ed. 2015.
- [10] I. K. Howard, "S is for Entropy. U is for Energy. What Was Clausius Thinking?," *J. Chem. Educ.*, vol. 78, no. 4, p. 505, Apr. 2001.
- [11] I. Dincer and Y. A. Cengel, "Energy, Entropy and Exergy Concepts and Their Roles in Thermal Engineering," *Entropy*, vol. 3, no. 3, pp. 116–149, Aug. 2001.
- [12] P. L. Corey, "NIST Guide to the SI, Chapter 4: The Two Classes of SI Units and the SI Prefixes," *NIST*, 28-Jan-2016. [Online]. Available: <https://www.nist.gov/pml/nist-guide-si-chapter-4-two-classes-si-units-and-si-prefixes>. [Accessed: 10-Jul-2017].
- [13] A. A. Minea, "Comparison of Environmentally Friendly Working Fluids for Organic Rankine Cycles," in *Advances in New Heat Transfer Fluids: From Numerical to Experimental Techniques*, CRC Press, 2017, pp. 377–426.
- [14] A. S. Karakurt, "A New Method for the Size and Performance Optimization of Thermal Systems: The Exergy Density," PhD Thesis, Graduate School of Natural and Applied Sciences, Yildiz Technical University, Istanbul, Turkey, 2018.
- [15] S. A. Klein, and F. L. Alvarado, EES-Engineering Equation Solver. 4406 Fox Bluff Rd. Middleton, WI 53562: F-Chart Software.
- [16] A. S. Karakurt, "Analysis of the part load conditions effects on turbine performance," Master Thesis, Graduate School of Natural and Applied Sciences, Yildiz Technical University, Istanbul, 2012.
- [17] A. S. Karakurt and U. Günes, "Performance Analysis of a Steam Turbine Power Plant at Part Load Conditions," *J. Therm. Eng.*, vol. 3, no. 2, pp. 1121–1128, 2017.