THERMODYNAMIC MODELING OF A PULSE TUBE REFRIGERATION SYSTEM

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ABSTRACT
Thermodynamic model of the pulse tube refrigeration (PTR) system has been developed based on the ideal gas behaviour to study the cooling effect at the cold end of the refrigerator. Compression and expansion processes of the gas column have been assumed to be isothermal. Mass flow in the regenerator has been evaluated through Ergun's law. Mass flow through the orifice and double inlet valve has been assumed a nozzle flow with a correction factor. Model predicted results have been validated with in-house experimental results as qualitative basis. Model predicted results in compression and expansion processes are also validated with that of the experimental data. Model predicted results are presented to understand the basic phenomenon for the refrigeration effect in various pulse tube refrigerators (BPTR, OPTR and DIPTR). Time duration for expansion process is more than the compression process in case of OPTR and DIPTR, which leads to lower pressure during the expansion and more cooling capacity obtained compared to the BPTR. A distinct comparison among three types of PTRs has been done based on the work done at the cold end. It has been clearly observed that a DIPTR shows better cooling capacity compared to OPTR or BPTR.

Keywords: Pulse Tube Refrigeration, Thermodynamic Model, Cooling Effect, Comparison

INTRODUCTION
Pulse tube refrigerator (PTR) has a great leverage over other cryo-coolers like Stirling and GM refrigerators due to the absence of moving components at the low temperature end. It has greater reliability due to no sources of mechanical vibration and no clearance seal between piston and cylinder. Moreover, it is a relatively low cost device with a simple and compact design. Though the invention of the pulse tube refrigeration had been in early 60’s [1], most of the development was done during last three decades [2,3]. Initially its operating principles were not fully understood. The oscillatory flow inside the pulse tube in the refrigerator and the associated thermodynamic processes are complex which is not similar like the other refrigeration systems [4,5]. As the level of understanding grows gradually, modifications and improved designs yield an improved efficiency. It has now become the most efficient cryocooler for a given size. It is also suitable for a wide variety of applications from civilian to government to military and from ground equipments to space systems. The moving displacer in the Stirling and G-M refrigerators is eliminated by a simple tube in the pulse tube refrigerator. The proper gas motion is achieved by the use of an orifice and a reservoir is used to store the gas during a half cycle. The reservoir volume is large enough that negligible pressure oscillation occurs during the oscillating flow. The oscillating flow through the orifice separates the heating and cooling effects just as the displacer does for the Stirling and G-M refrigerators. The orifice pulse tube refrigerator (OPTR) operates ideally with adiabatic compression and expansion. The function of the pulse tube is to insulate the processes at its two ends. Gas in the middle portion of the pulse tube never leaves the pulse tube and thus forming a temperature gradient that insulates the two ends from each other [6]. The pulse tube is the unique component in this refrigerator. Though it is an open tube, the thermo-hydrodynamic processes involved are extremely complex and not well understood or successfully modelled [6]. Its function is to transmit hydrodynamic power in an oscillating gas system from one end to the other end across a temperature gradient with a minimum of power dissipation and entropy generation. The basic pulse-tube refrigerator (BPTR) consists of a pressure-wave generator, a regenerator, and a tube only. There are no orifices of any kind. The operation is based on the thermal interaction between the gas and the tube wall. Performance of the BPTR is improved by adding an orifice and a buffer commonly known as Orifice pulse tube refrigerator (OPTR), but mass flow through the regenerator increases leading to a degradation of regenerator performance. In the double-inlet PTR (DIPTR), the hot end of the pulse tube is connected with the entrance (hot end) of the regenerator by a valve. The double inlet is a bypass.
for the regenerator and the tube, and it reduces the dissipation in the regenerator. Many researchers have developed models to predict the performance of the PTRs based on some empirical parameters, which differs from author to author. Thermodynamic analysis of the pulse tube refrigeration system is not new. But for the purpose of analysis, researchers have used different schemes for dividing the full pulse tube refrigerator into subsystems. Zhu et al. [3] proposed to divide the pulse tube section into three subsystems, the middle section of the tube separating the cold and the hot volumes. In their approach, no heat exchange occurs in the mid-section, and that pressure variation induces temperature swing in the gas. Liang et al. [7] developed a model to understand the working mechanism of three common types of pulse tube refrigerator by analyzing the thermodynamic behaviour of gas elements as adiabatic process in the pulse tube. They assumed the pressure variation inside the pulse tube to be symmetric at the time of compression and expansion. For the simplicity of calculations they took it as trapezoidal variation with time. Recently, CFD models are more popular because of its complexity geometry and it takes more computation time compare to thermodynamic models [8,9,10] developed a two-dimensional computational fluid dynamic (CFD) simulation of a GM type double inlet pulse tube refrigerator (DIPTR), operating under a variety of thermal boundary conditions analyzed by using commercial CFD package to model the oscillating flow inside a pulse tube refrigerator to predict the performance. Chen et al. [11] studied the thermodynamic cycles in an instance tube pulse tube refrigerator (ITPTR) by means of CFD techniques. They observed that gas parcels working in different parts of ITPTR undergo different thermodynamic cycles. A one-dimensional finite volume discretization model is proposed implemented in a computer program for the modeling of a Stirling type Pulse Tube Cryocoolers (PTC) to study the performance. Temperature dependency of thermo-physical properties are taken into account between the working fluid and the solid parts, and heat conductions in the gas and solid [12]. A detailed time-dependent axisymmetric computational fluid dynamic (CFD) model of the OPTR has also been developed to predict the performance by Antao and Farouk [13]. An improved representation of heat transfer in the porous media is achieved by employing a thermal non-equilibrium model to couple the gas and solid (porous media) energy equations. Dai et al. [14] developed model to investigate the three dimensional oscillating flow and heat transfer in the pulse tube and heat exchangers of a pulse tube refrigerator. Zhang et al. [9] developed a linear model for theoretical analyses for the orifice and double-inlet PTCs that indicates the cooling performance can be improved by introducing the cold reservoir. Park et al. [15] also analyzed to identify loss mechanisms with the simple numerical computation (linear model) followed by Zhang et al. [16] which considers the dynamic characteristics of the cold linear compressor with thermo-hydraulic governing equations for each of sub components of the PTR. All the mass flows and pressure waves were assumed to be sinusoidal. But practically it never happens.

In the present work, the variation of stroke volume of the compressor cylinder is considered sinusoidal in nature. In this model, the compression and expansion processes are taken as isothermal and thus it is reversible observed by Neveu and Babo [17]. In addition, it shows a higher efficiency than the adiabatic or any other model of the pulse tube. They assumed that each volume is subjected to several complex processes. These are due to heat exchange in the associated heat exchanger, surface heat pumping in the tube and the mixing of the gas. They expressed all these phenomena through lumped parameters, considering isothermal heat exchange. They developed the model only for Stirling type orifice pulse tube refrigerator. In the present work, an isothermal model has been developed that has ability to predict the performance any basic PTRs and a comparison among them has been done.

**MODEL DESCRIPTIONS**

A pulse tube refrigeration system is a closed system that uses an oscillating pressure (typically produced by a compressor) at one end to generate an oscillating gas flow in the rest of the system. The schematic arrangement of a Stirling type pulse tube refrigerator (PTR) system is shown in Figure 1.

The pulse tube device is divided into six open subsystems. Three of them exchange work, heat and mass with the surroundings (compressor, cold and hot volumes), while the others exchange mass only (regenerator, double inlet valve and orifice reservoir). It has been assumed that all heat exchanges occur at a constant temperature, and that temperature of all subsystems exchanging heat are equal to those of the heat reservoirs. Another condition is that mechanical equilibrium is realized in each part of the device. These conditions lead to the model presented in Figure 2.
The system described in the Figure 2 consists of six opened subsystems.

- Isothermal compressor at $T_{cp}$, exchanging mechanical work $\dot{W}_{cp}$ and heat $\dot{Q}_{cp}$ with the surroundings at temperature $T_h$ and mass flow rate $\dot{m}_{cp}(\dot{m}_{rg} + \dot{m}_{di})$, where $\dot{m}_{rg}$ with the regenerator and $\dot{m}_{di}$ with the double inlet valve.

- Regenerator, exchanging mass rate $\dot{m}_{rg}$ at $T_{cp}$ with the compressor $\dot{m}_c$ and at $T_c$ with the pulse tube.

- Cold volume in the pulse tube at $T_c$, exchanging heat $\dot{Q}_c$ with the surrounding at temperature $T_c$, mass $\dot{m}_c$ with the regenerator, and mechanical work $\dot{W}_c$ with the hot volume.

- Hot volume in the pulse tube at $T_h$, exchanging mechanical work $\dot{W}_h$ with the cold volume, heat $\dot{Q}_h$ with the surrounding at temperature $T_{cp}$ and mass $\dot{m}_h(\dot{m}_o - \dot{m}_{di})$ with the reservoir via orifice and DI valve.

- Adiabatic orifice and reservoir, exchanging mass $\dot{m}_o$ with hot volume.

- Double inlet valve exchanging mass $\dot{m}_{di}$ with compressor and hot volume.

Each subsystem has two states, viz. gas and wall. Conservation of mass and energy has been applied to the six subsystems with the following assumptions:

- Ideal perfect gas,
- Uniform temperature in each subsystem,
- Mechanical equilibrium in compressor and pulse tube, ideal regenerator, i.e. front wave moving back and forth during a cycle,
- Piston movement in the pulse tube.
- Negligible gas volume in the regenerator compared to compressor, tube and reservoir.
- Negligible pressure losses due to pipe and bends.
GOVERNING EQUATIONS

Isothermal opened variable volume:

\[ \frac{dm}{dt} = \dot{m} \]  \hspace{1cm} (1)

\[ \frac{dU}{dt} = \dot{w} + \dot{q} + \dot{m} h \]  \hspace{1cm} (1)

Assuming gas in the pulse tube as ideal gas

\[ PV = mRT \]  \hspace{1cm} (2)

Equation (2) can be written as

\[ dm = m \left( \frac{dP}{P} + \frac{dV}{V} - \frac{dT}{T} \right) \]  \hspace{1cm} (3)

Change of the internal energy of the gas inside the system volume can be written as [17],

\[ dU = mc_v dT + u dm \]  \hspace{1cm} (4)

As temperature has been assumed constant, \( dT = 0 \), so that Equations (4 and 5) can be simplified as

\[ dm = m \left( \frac{dP}{P} + \frac{dV}{V} \right) = \frac{1}{RT} \left( V \dot{d}P + P \dot{d}V \right) \]  \hspace{1cm} (5)

\[ dU = u dm \]

Substituting Eq. (6) in Eq. (2), and assuming mechanical equilibrium in the system volume,

\[ \frac{dP}{dt} = \frac{1}{V} \left( \dot{m} RT - P \frac{dV}{dt} \right) \]

\[ \dot{q} = P \frac{dV}{dt} - \dot{m} \left( PV \right) = P \frac{dV}{dt} - \dot{m} RT \]  \hspace{1cm} (6)

\[ \dot{w} = -P \frac{dV}{dt} \]

For Isothermal Compressor

Applying the above sets of equations to the compressor, Eq. (7) becomes
\[
\frac{dP_{cp}}{dt} = \frac{1}{V_{cp}} \left( -\dot{m}_{cp}RT_{cp} - P_{cp} \frac{dV_{cp}}{dt} \right),
\]
\[
\dot{q}_{cp} = P_{cp} \frac{dV_{cp}}{dt} + \dot{m}_{cp}RT_{cp},
\]
\[
\dot{w}_{cp} = -P_{cp} \frac{dV_{cp}}{dt}
\] (7)

**For Pulse tube**

Similarly to that in compressor, the pulse tube flow has been assumed to be a piston like flow. The pulse tube has been divided into two distinct volumes, one for cold volume, \(V_c\), and the other for the hot volume \(V_h\) at uniform temperature to ensure the reversibility of the model.

For cold volume,
\[
\frac{dP_c}{dt} = \frac{1}{V_c} \left( \dot{m}_cRT_c - P_c \frac{dV_c}{dt} \right),
\]
\[
\dot{q}_c = P_c \frac{dV_c}{dt} - \dot{m}_cRT_c,
\]
\[
\dot{w}_c = -P_c \frac{dV_c}{dt}
\] (8)

and for hot volume,
\[
\frac{dP_h}{dt} = \frac{1}{V_h} \left( -(\dot{m}_o - \dot{m}_{di})RT - P_h \frac{dV_h}{dt} \right),
\]
\[
\dot{q}_h = P_h \frac{dV_h}{dt} + (\dot{m}_o - \dot{m}_{di})RT_h,
\]
\[
\dot{w}_h = P_h \frac{dV_h}{dt}
\] (9)

Mechanical equilibrium yields
\[
\dot{w}_c + \dot{w}_h = 0
\] (10)

Combining Eqs. (8) to (10) leads to the following set of equations, describing the pulse tube:
\[
\frac{dP_c}{dt} = \frac{1}{V_c} \left( \dot{m}_cRT_c - (\dot{m}_o - \dot{m}_{di})RT_h \right),
\]
\[
\frac{dX_c}{dt} = \frac{1}{P_iV_c} \left( \dot{m}_cRT_c \left(1 - X_c\right) + (\dot{m}_o - \dot{m}_{di})RT_h X_c \right),
\]
\[
\dot{q}_c = P_iV_c \frac{dX_c}{dt} - \dot{m}_cRT_c,
\]
\[
\dot{q}_h = -P_iV_c \frac{dX_c}{dt} + (\dot{m}_o - \dot{m}_{di})RT_h,
\] (11)

where \(X_c = \frac{V_c}{V_t}\) is the dimensionless cold volume.
Orifice plus reservoir:

Applied to the reservoir, Eq. (7) becomes

\[
\frac{dP_r}{dt} = \frac{1}{V_r} (-m_o RT_h)
\]
\[
\dot{q}_r = m_o RT_h
\]
\[
\dot{w} = 0,
\]

(12)

Mass flow in the regenerator has been evaluated through Ergun’s law. Mass flow through the orifice and double inlet valve has been assumed a nozzle flow with a correction factor [18].

Change in Compressor Volume:

Sinusoidal variation has been taken for the compressor volume

\[
V_{cp}(t) = V_o + \frac{V_s}{2} [1 + \sin(2\pi ft)]
\]

(13)

SOLVING METHODOLOGY

It has been assumed that temperatures are constant and uniform at short time scale. Then the whole device can be described by the following four properties: \( P_{cp}, P_r, P_t \) and \( X_t \) varying in time according to the differential equations described above.

\[
\frac{dP_{cp}}{dt} = \frac{1}{V_{cp}} \left( -\dot{m}_{cp} RT - P_{cp} \frac{dV_{cp}}{dt} \right),
\]
\[
\frac{dP_t}{dt} = \frac{1}{V_t} \left( \dot{m}_h RT_t - (\dot{m}_h - \dot{m}_{di}) RT_h \right),
\]
\[
\frac{dX_t}{dt} = \frac{1}{P V_t} \left( \dot{m}_h RT_t (1 - X_t) + (\dot{m}_o - \dot{m}_{di}) RT_h X_t \right),
\]
\[
\frac{dP_r}{dt} = \frac{1}{V_r} (-\dot{m}_o RT_h)
\]

(14)

These are all time varying ordinary differential equations and need only one initial condition for each to solve. From equation (13), the following initial conditions are derived as shown in figure 4.

*Figure 4. Variation of volume with time or angle*

At \( t=0 \), it can be assumed that \( \theta = \omega t = 2\pi ft = 0 \)
This means that the piston will be at middle position when the compressor is at average compressor pressure. Initially the whole system is in equilibrium state, the pressure at the pulse tube and the reservoir will be at same pressure. As the flow in the pulse tube has been assumed as piston like flow, the gas piston will be at the middle position. Therefore,

\[ X_c(t) = X_c(0) = \frac{V_c(0)}{V_t} = 0.25 \]  

Fourth order Runge-kutta Method has been used to calculate \( P_{cp}, P_t, P_r \) and \( X_t \) with respect to time. Design data of the model are listed in Table 1.

### Table 1. Design data for the model

<table>
<thead>
<tr>
<th>Components</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor</td>
<td>Dead volume(=V_o= 0.13 \times 10^{-3})m(^3)</td>
</tr>
<tr>
<td></td>
<td>Swept volume(=V_s= 0.5 \times 10^{-3})m(^3)</td>
</tr>
<tr>
<td>Regenerator</td>
<td>Length(=L_{reg}=0.3)m</td>
</tr>
<tr>
<td></td>
<td>Diameter(=D_{reg}=0.032)m</td>
</tr>
<tr>
<td></td>
<td>Porosity(=0.7)</td>
</tr>
<tr>
<td></td>
<td>Hydraulic diameter: 0.04mm</td>
</tr>
<tr>
<td>Pulse tube</td>
<td>Length(=L_r=0.8)m</td>
</tr>
<tr>
<td></td>
<td>Diameter(=D_r=0.02)m</td>
</tr>
<tr>
<td></td>
<td>Volume(=V_r=0.00025)m(^3)</td>
</tr>
<tr>
<td>Orifice</td>
<td>Diameter(=1)mm</td>
</tr>
<tr>
<td>DI valve</td>
<td>Diameter(=1)mm</td>
</tr>
<tr>
<td>Reservoir</td>
<td>Volume(=0.007)m(^3)</td>
</tr>
<tr>
<td>Average pressure</td>
<td>10 bar</td>
</tr>
<tr>
<td>Frequency</td>
<td>2 Hz</td>
</tr>
<tr>
<td>Cold end temperature</td>
<td>100 K</td>
</tr>
<tr>
<td>Hot end temperature</td>
<td>300 K</td>
</tr>
<tr>
<td>Helium gas at 10 bar and 200 K temperature</td>
<td>Viscosity= ( \mu=15.21 \times 10^{-6})Ns/m(^2)</td>
</tr>
<tr>
<td></td>
<td>Density= ( \rho=2.389)Kg/m(^3)</td>
</tr>
<tr>
<td></td>
<td>( c_p=5193.0)J/Kg K</td>
</tr>
<tr>
<td></td>
<td>( R=2074.6)J/Kg K</td>
</tr>
<tr>
<td></td>
<td>( K=1.66)</td>
</tr>
</tbody>
</table>

### RESULTS AND DISCUSSION

The present model has predicted the pressure variation in the compressor, pulse tube and reservoir (constant) with time in case of BPTR, OPTR and DIPTR configurations which have been presented as shown in Figure 5–7. In the present study, an effort has been made for a qualitative comparison of pressure variation at different section of the pulse tube between the experimental results and model predicted results. It is noted that developed model is based on Stirling type arrangement and experimental studies are carried out with GM type. Experimental details are given in the reference, Roy [18]. Pressure variation at regenerator inlet, pulse tube and reservoir with respect to time in a GM type pulse tube refrigerator has been shown in Figure 8. In GM type pulse tube refrigerator, a rotary valve is used to switch system pressure with high pressure (HP) and low pressure (LP) lines of the compressor to create compression and expansion process in the pulse tube. It is observed from both the results that although, there have some quantitative difference between experimental and model prediction, a distinct phase relationship among the pressures are observed in both experimental results and model predicted results as shown in Table 2. Pulse tube refrigerator has been operated at a frequency of 2 Hz. Model predicted results of compression and expansion duration (time) are well validated with the experimental data. Variations of maximum and minimum pressure variation in compressor and pulse tube are due to its inherent types of the refrigerator.
Table 2. Data for model validation

<table>
<thead>
<tr>
<th>Data Observed</th>
<th>Experimental data (GM Type)</th>
<th>Model Prediction (Stirling Type)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression duration</td>
<td>0.19s</td>
<td>0.21s</td>
</tr>
<tr>
<td>Expansion duration</td>
<td>0.31s</td>
<td>0.29s</td>
</tr>
</tbody>
</table>

Figure 5. Pressure variation of compressor ($P_{cp}$), pulse tube ($P_t$) and reservoir ($P_r$) with time in case of BPTR.

Figure 6. Pressure variation of compressor ($P_{cp}$), pulse tube ($P_t$) and reservoir ($P_r$) with time in case of OPTR.

Figure 7. Pressure variation of compressor ($P_{cp}$), pulse tube ($P_t$) and reservoir ($P_r$) with time in case of DIPTR.
In Figure 5, pressure variation at the compressor, pulse tube and reservoir are presented in a frequency of 2 Hz. Time duration of compression and expansion are same, 24s, since there is no reservoir, whole masses are compressed and expanded in a closed system. Maximum pressure rise at the compressor and pulse tube are 13.9 bar and 12.6 bar, respectively whereas minimum pressure during expansion are 7.8 bar and 8.4 bar, respectively. The average pressure difference (10bar) in expansion process is less compared to the compression process. It means that BPTR are not capable to reach minimum pressure or temperature at the cold end of the pulse tube. On the other end, in OPTR and DIPTR, time taken for expansion process is more (~0.29s) compared to compression process (~0.21s). It leads to increase more pressure different during expansion process. Minimum pressure at the compressor and pulse tube during expansion are 7.1 bar and 7.7 bar, respectively. So, OPTR and DIPTR are capable to reach minimum temperature compared to that in BPTR.

With this above qualitative validation of the developed model, a comparison among the BPTR, OPTR, and DIPTR have been made based on the cold end work done produced in different types of PTRs. Volume variation of compressor (CP), cold end (CE) and hot end (HE) with time in case of BPTR are presented in Figure 9. Now compressor (CP), cold end (CE) and hot end (HE) work-done in a cyclic operation in case of BPTR has been presented in Figure 10 by combining Figure 5 and 9. It has been observed from Figure 10 that no PV work occurs at the cold end as well as at the hot end volume. It indicates that there is no cooling or heating effect in case of BPTR. This result supports the fact that surface heat pumping mechanism is responsible for cooling in BPTRs. But in case of Figure 11 and 12, a certain amount of PV work done have been observed due to adequate phase relationship between pressure and mass flow rate occurs in OPTR and DIPTR. Similar observations have been made for gas parcels in the pulse tube by Chen et al. [11] and Qui et al. [19].
A Comparison among the BPTR, OPTR and DIPTR has been made based on an enlarged scale cold end work-done shown in Figure 13. BPTR shows no PV area, i.e. compression and expansion follow in the same path. But OPTR and DIPTR show their PV areas, i.e. having a cooling work. But DIPTR has performed the
compression and expansion work within a shorter range compared to OPTRs with higher amplitude. That means DIPTR configuration provides more cooling capacity than that of the OPTR. 

![Diagram](image_url)

**Figure 13.** Cold end work done in case of BPTR, OPTR and DIPTR

CONCLUSIONS

To study the cooling effect at the cold-end of the Stirling type pulse tube refrigerator, a thermodynamic model has been proposed. This model has ability to predict pressure variations in the compressor, pulse tube and have been validated with in-house experimental results as a qualitative basis. It is observed from both the cases that though there have some quantitative difference between them, but a distinct time lag relationship among the pressures are observed in both the experimental results and model predicted results. However, model predicted results in compression and expansion are well validated with the experimental data. From the present model, it is understood the basic phenomenon which is responsible for the refrigeration effect in different types of pulse tube refrigerators (BPTR, OPTR and DIPTR). Time duration for expansion process is more than the compression process in case of OPTR and DIPTR, which leads to lower pressure during expansion and more cooling effect compared to the BPTR. A distinct comparison among three types of PTRs has been clearly shown for the cold end work done and DIPTR shows more cold-end PV–area compared to OPTR. Therefore, DIPTR shows always a better cooling capacity compared to OPTR and BPTR

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REFERENCES


